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Self-consistent kaon and pion dynamics in the nuclear medium

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2. Projector algebra for in-medium scattering
3. Numerical results: kaons and hyperons in the medium
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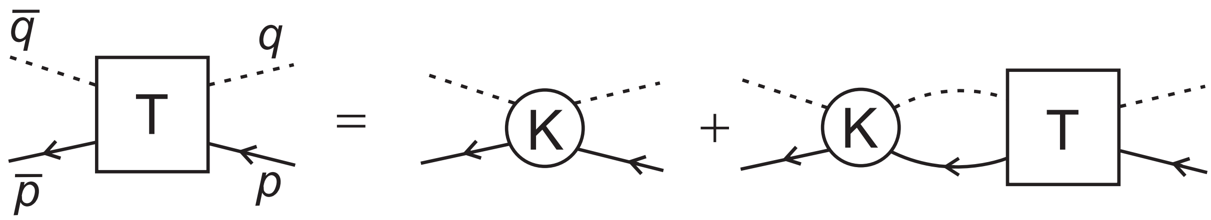
1. Description of kaon-nucleon scattering in vacuum

The scattering amplitude T is given as a solution of the Bethe-Salpeter equation, which iterates the interaction kernel K

$$T = K + K\,G\,T$$



The Bethe-Salpeter equation

$$\begin{aligned} T(\bar{k},k;w) &= K(\bar{k},k;w) + \int \frac{d^4 l}{(2\pi)^4} K(\bar{k},l;w) \\ &\quad \cdot G(l;w)\,T(l,k;w), \\ G(l;w) &\equiv -iS_N(\frac{1}{2}w + l)\,D_{\bar{K}}(\frac{1}{2}w - l), \end{aligned}$$

with:

$$w \equiv p + q = \bar{p} + \bar{q}, \quad k \equiv \frac{1}{2}(p - q), \quad \bar{k} \equiv \frac{1}{2}(\bar{p} - \bar{q}).$$

S. Weinberg suggested chiral expansion of $K(\bar{k}, l; w)$

to chiral order Q^3 the s-, p- and d-waves appear:

$$
\begin{aligned}
K(\bar{k}, k; w) &= g_0^{(+)}(s) Y_0^{(+)}(\bar{q}, q; w) \text{ (s-wave, } J = 1/2) \\
&+ g_0^{(-)}(s) Y_0^{(-)}(\bar{q}, q; w) \text{ (p-wave, } J = 1/2) \\
&+ g_1^{(+)}(s) Y_1^{(+)}(\bar{q}, q; w) \text{ (p-wave, } J = 3/2) \\
&+ g_1^{(-)}(s) Y_1^{(-)}(\bar{q}, q; w) \text{ (d-wave, } J = 3/2)
\end{aligned}
$$

where $s \equiv w^2$, and

$$
\begin{aligned}
Y_0^{(\pm)}(\bar{q}, q; w) &= \frac{1}{2} \left(\frac{\not{w}}{\sqrt{w^2}} \pm 1 \right) , \\
Y_1^{(\pm)}(\bar{q}, q; w) &= \frac{3}{2} \left(\frac{\not{w}}{\sqrt{w^2}} \pm 1 \right) \left(\frac{(\bar{q} \cdot w)(w \cdot q)}{w^2} - (\bar{q} \cdot q) \right) \\
&\quad - \frac{1}{2} \left(\not{\bar{q}} - \frac{w \cdot \bar{q}}{w^2} \not{w} \right) \left(\frac{\not{w}}{\sqrt{w^2}} \mp 1 \right) \left(\not{q} - \frac{w \cdot q}{w^2} \not{w} \right) .
\end{aligned}
$$

Solution for T (M. Lutz and E.E. Kolomeitsev, Nucl. Phys. A700 (2002) 193):

$$
\begin{aligned}
T(\bar{k}, k; w) &= t_0^{(+)}(s) Y_0^{(+)}(\bar{q}, q; w) \\
&+ t_0^{(-)}(s) Y_0^{(-)}(\bar{q}, q; w) \\
&+ t_1^{(+)}(s) Y_1^{(+)}(\bar{q}, q; w) \\
&+ t_1^{(-)}(s) Y_1^{(-)}(\bar{q}, q; w)
\end{aligned}
$$

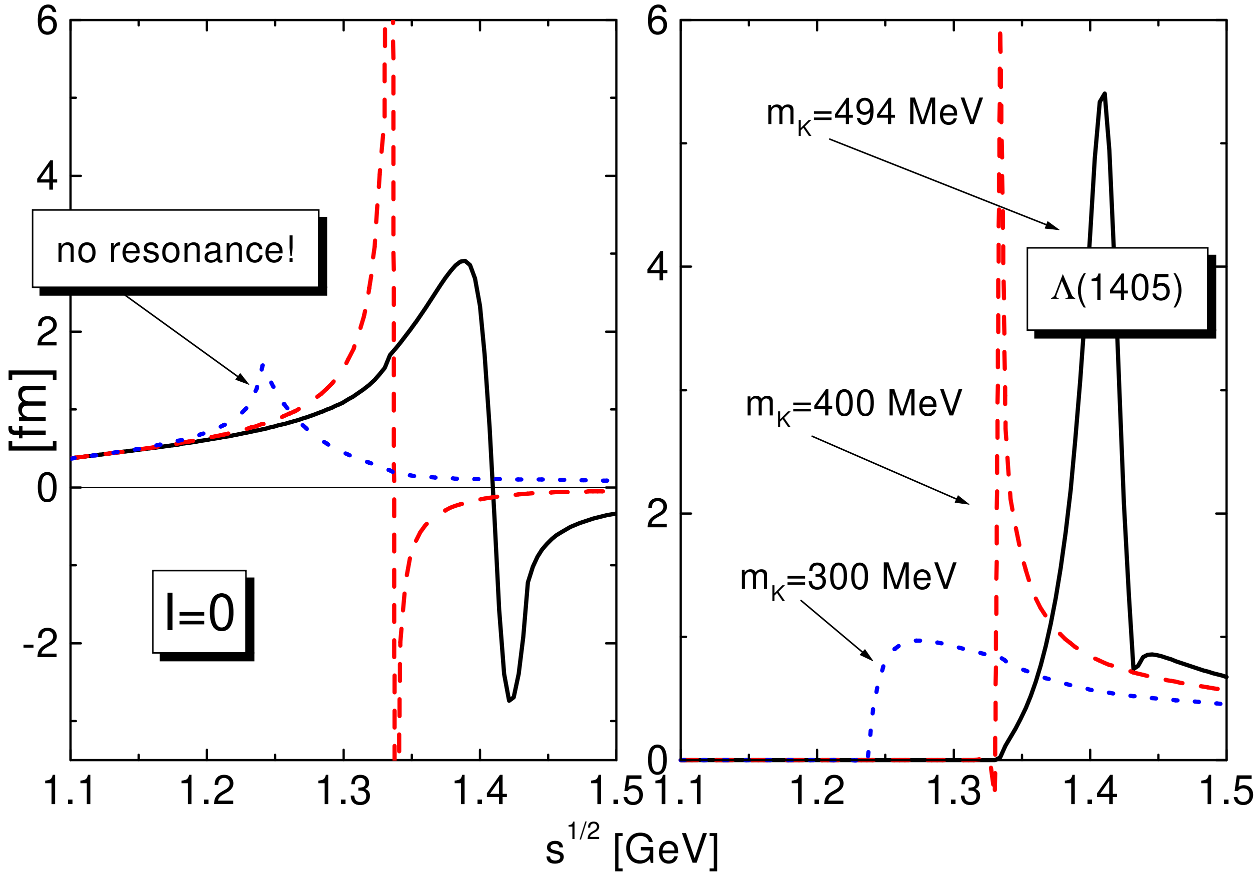
where the reduced amplitudes are

$$
t_n^{(\pm)}(s) = \frac{g_n^{(\pm)}(s)}{1 - g_n^{(\pm)}(s)\, J_n^{(\pm)}(s)},
$$

with $J_n^{(\pm)}$ being the (regularized) loop integrals, defined by

$$J_n^{(\pm)}(s)\, Y_n^{(\pm)}(\bar{q}, q; w) =$$
$$-i \int \frac{d^4 l}{(2\pi)^4} Y_n^{(\pm)}(\bar{q}, l; w) G_N(w-l) D_K(l) Y_n^{(\pm)}(l, q; w)$$

A fit to low-energy meson-nucleon scattering data (Lutz, Kolomeitsev) determines the coefficients in the chiral expansion of the kernel K



2. Projector algebra for in-medium scattering

$\hat{T} = \hat{K} + \hat{K}\,\hat{G}\,\hat{T}$, (BS equation in the medium)

but we assume $\hat{K} = K$

Approach: Do not base in-medium calculation on the kernel K (which is model dependent), but on T

Using $T = K + KGT$

$$\hat{T} = T + T\,\Delta G\,\hat{T}, \quad \Delta G \equiv \hat{G} - G$$

Question: How can we expand $\hat{T}$ in terms of reduced amplitudes similar to $t_n^{(\pm)}(s)$?
Answer:

$$\begin{aligned}\hat{T}(\bar{k}, k; w_0, \boldsymbol{w}) \;&=\; \hat{T}(w_0, \boldsymbol{w}) + \hat{T}^{\nu}(w_0, \boldsymbol{w}) q_\nu \\ &\quad + \bar{q}_\mu \bar{\hat{T}}^{\mu}(w_0, \boldsymbol{w}) + \bar{q}_\mu \hat{T}^{\mu\nu}(w_0, \boldsymbol{w}) q_\nu,\end{aligned}$$

If we include only waves with $J = 1/2$:

$$\begin{aligned}\hat{T}(w_0, \boldsymbol{w}) = a_P^{(+)}(w_0, \boldsymbol{w}) P_+ + a_P^{(-)}(w_0, \boldsymbol{w}) P_- \\ + a_Q^{(+)}(w_0, \boldsymbol{w}) Q_+ + a_Q^{(+)}(w_0, \boldsymbol{w}) Q_+,\end{aligned}$$

the other terms are zero, where:

$$P_\pm \equiv \frac{1}{2}\left(1 \pm \frac{\not{w}}{\sqrt{s}}\right) = \pm Y_0^{(\pm)}$$

$$Q_\pm \equiv \frac{\sqrt{s}}{\sqrt{s - w_0^2}} P_\pm \gamma_0 P_\pm$$

Multiplication rules:

$$P_\pm P_\pm = P_\pm \ , \quad P_\pm P_\mp = 0 \ , \quad Q_\pm Q_\mp = P_\pm \ ,$$

$$Q_\pm^2 = 0 \quad , P_\pm Q_\pm = Q_\pm = Q_\pm P_\mp .$$

With $J = 3/2$ waves:

$$\bar{\hat{T}}^\mu(w_0, \boldsymbol{w}) = \sum_{j=1}^{6} \left(\bar{\mathcal{M}}_j^{(p)} \bar{P}_j^\mu + \bar{\mathcal{M}}_j^{(q)} \bar{Q}_j^\mu \right) ,$$

$$\hat{T}^\mu(w_0, \boldsymbol{w}) = \sum_{j=1}^{6} \left(\mathcal{M}_j^{(p)} P_j^\mu + \mathcal{M}_j^{(q)} Q_j^\mu \right) ,$$

$$\hat{T}^{\mu\nu}(w_0, \boldsymbol{w}) = \sum_{j=-1}^{18} \left(\mathcal{M}_{j,t}^{(p)} P_j^{\mu\nu} + \mathcal{M}_{j,t}^{(q)} Q_j^{\mu\nu} \right) .$$

where $\mathcal{M}_{j[,t]}^{(p,q)}(w_0, \boldsymbol{w})$ are the reduced amplitudes

Construction of projectors – introduce orthogonal Lorentz vectors

In vacuum we already had

$$Y_1^{(\pm)} = \mp 3 \bar{q}_\mu \left((g^{\mu\nu} - \hat{w}^\mu \hat{w}^\nu) P_\pm - V^\mu P_\mp V^\nu \right) q_\nu$$

where:

$$\hat{w}_\mu \equiv \frac{w_\mu}{\sqrt{s}} , \qquad V_\mu \equiv \frac{1}{\sqrt{3}} (\gamma_\mu - \frac{\not{w}}{w^2} w_\mu) \ , \quad V_\mu V^\mu = 1,$$

$$\widehat{w}^2 = 1, \quad \widehat{w}\cdot V = 0.$$

Define $\tilde{w} \equiv (w_0, \mathbf{0})$ and

$$X_\mu \equiv \frac{(w\cdot\tilde{w})\,w_\mu - w^2\,\tilde{w}_\mu}{\sqrt{w^2}\sqrt{(w\cdot\tilde{w})^2 - w^2\,\tilde{w}^2}}\ , \ X\cdot w = 0, \quad X^2 = -1,$$

It is further convenient to introduce:

$$R_\mu = \frac{i}{\sqrt{3}}\left(Q_+ + Q_-\right)V_\mu + X_\mu\ ,$$

$$L_\mu = \frac{i}{\sqrt{3}}V_\mu\left(Q_+ + Q_-\right) + X_\mu\ .$$

which satisfy

$$R_\mu\,V^\mu = 0 = V^\mu\,L_\mu, \quad R_\mu\,L^\mu = -\frac{2}{3}.$$

$$\bar{P}_1^\mu \equiv V^\mu P_+, \qquad \bar{P}_2^\mu \equiv \widehat{w}^\mu P_+, \qquad \bar{P}_3^\mu \equiv \widehat{L}^\mu P_+,$$

$$\bar{P}_4^\mu \equiv V^\mu P_-, \qquad \bar{P}_5^\mu \equiv \widehat{w}^\mu P_-, \qquad \bar{P}_6^\mu \equiv \widehat{L}^\mu P_-.$$

The $\bar{Q}_j^\mu$ projectors are defined analogously

Another set is:

$$P_1^\mu \equiv P_+V^\mu, \qquad P_2^\mu \equiv P_+\widehat{w}^\mu, \qquad P_3^\mu \equiv P_+\widehat{R}^\mu,$$

$$P_4^\mu \equiv P_-V^\mu, \qquad P_5^\mu \equiv P_-\widehat{w}^\mu, \qquad P_6^\mu \equiv P_-\widehat{R}^\mu.$$

The projectors with 2 Lorentz-vector indices are defined through:

$$P_i^{\mu\nu} \sim \bar{P}_j^{\mu}\, P_k^{\nu}, \;\; \bar{P}_j^{\mu}\, Q_k^{\nu}$$

It is not difficult to write down 2 projectors which are orthogonal (from left and right) to all $\bar{P}_j^{\mu}\, P_k^{\nu},\ \bar{P}_j^{\mu}\, Q_k^{\nu}$:

$$P_{0,\pm}^{\mu\nu} = (g^{\mu\nu} - \widehat{w}^{\mu}\widehat{w}^{\nu})P_{\pm} - V^{\mu}P_{\mp}V^{\nu} - L^{\mu}P_{\pm}R^{\nu}$$

Question: Are there another 2?

Answer: Yes! By construction:

$$P_{0,\pm}^{\mu\alpha}\, Q_{\pm}\, P_{0,\mp}^{\alpha\nu}$$

is orthogonal from left and right to all $\bar{P}_j^{\mu}\, P_k^{\nu},\ \bar{P}_j^{\mu}\, Q_k^{\nu}$ and is nontrivial

Integrals containing ΔG can be expanded in similar way, for example:

$$\int \frac{d^4 l}{(2\pi)^4} l^{\mu} l^{\nu} \Delta G(l - \frac{1}{2}w; w_0, \boldsymbol{w}) =$$
$$\sum_{j=-1}^{18} \left(\Delta J_{j,t}^{(p)}\, P_j^{\mu\nu} + \Delta J_{j,t}^{(q)}\, Q_j^{\mu\nu} \right),$$

and also the vacuum amplitude T (with a lot of expansion coefficients equal to zero)

The BS equation reduces to a set of algebraic equations (in terms of reduced amplitudes), whose solution is:

$$\hat{T} = (1 - T\,\Delta G)^{-1}\,T$$

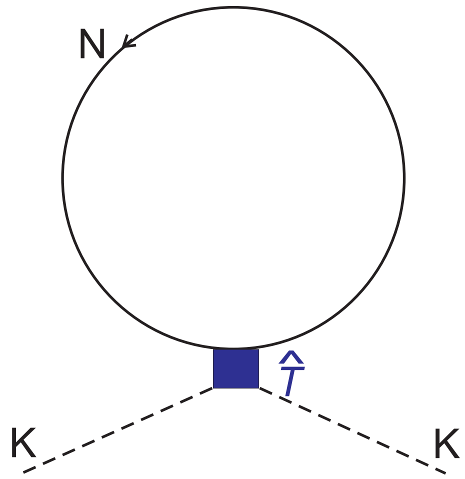
How to implement the projector multiplication table?

Through matrix multiplication (assigning the matrix elements in a right way)

To each point given by $w_0, |\boldsymbol{w}|$ we assign a 10×10 matrix (2×2 + 8×8), corresponding to $\hat{T}$, and another one corresponding to ΔG

Actually the 8×8-as matrix reduces to a 4×4 (other elements are zero)

The kaon self energy is calculated from the in-medium scattering amplitude (where the loop integrals contain the kaon propagator with the same self energy):



$$\Pi_{\bar{K}}(q,u) \;=\; -2\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,i\Delta S_N(p,u) \cdot\bar{\hat{T}}_{\bar{K}N}\left(\frac{p-q}{2},\frac{p-q}{2};p+q,u\right),$$

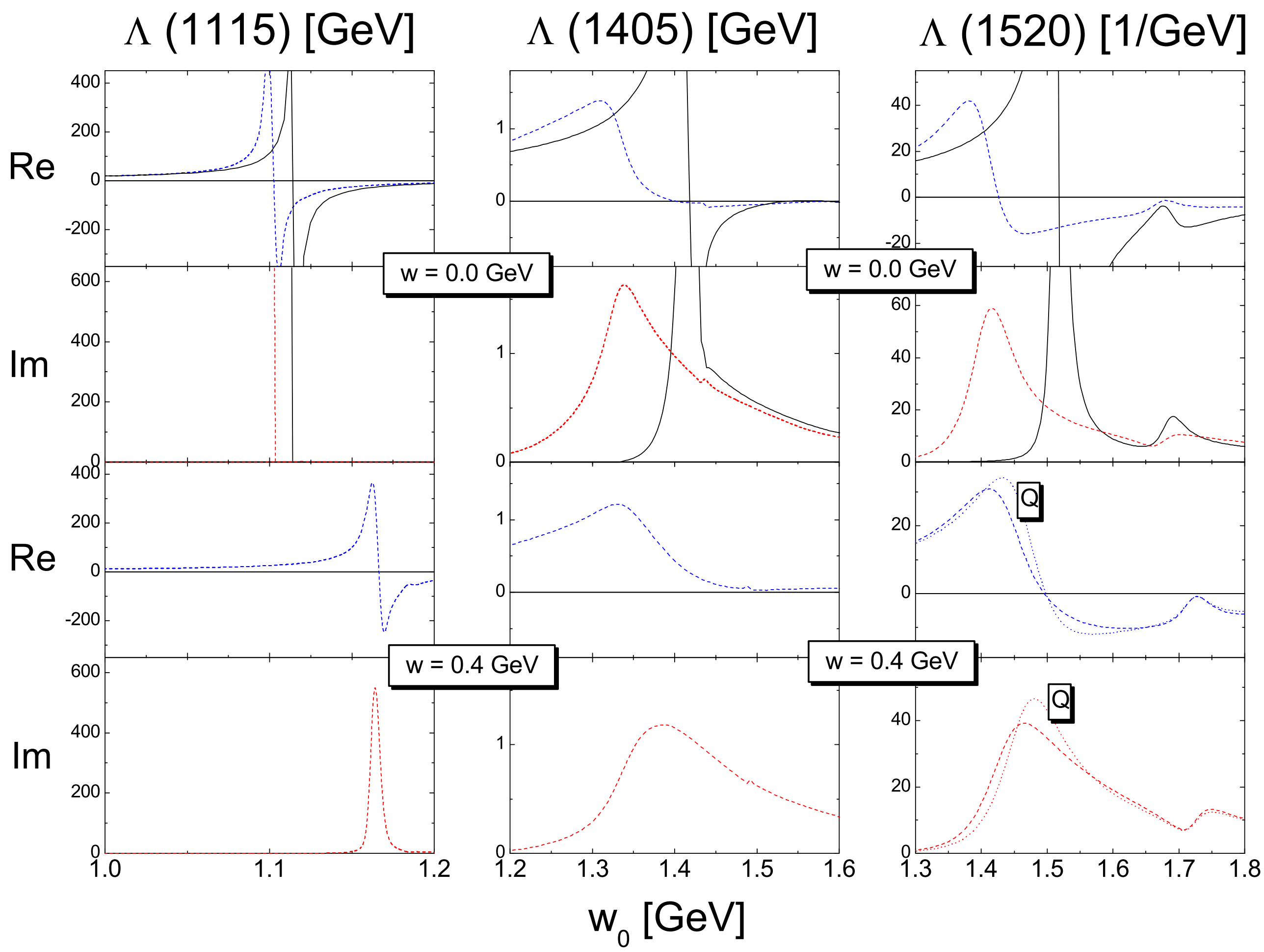
$$\bar{\hat{T}}_{\bar{K}N} = \frac{1}{4}\hat{T}^{(I=0)}_{\bar{K}N} + \frac{3}{4}\hat{T}^{(I=1)}_{\bar{K}N}.$$

After calculating the trace the integrand is left as a function of reduced amplitudes (with certain kinematical factors)

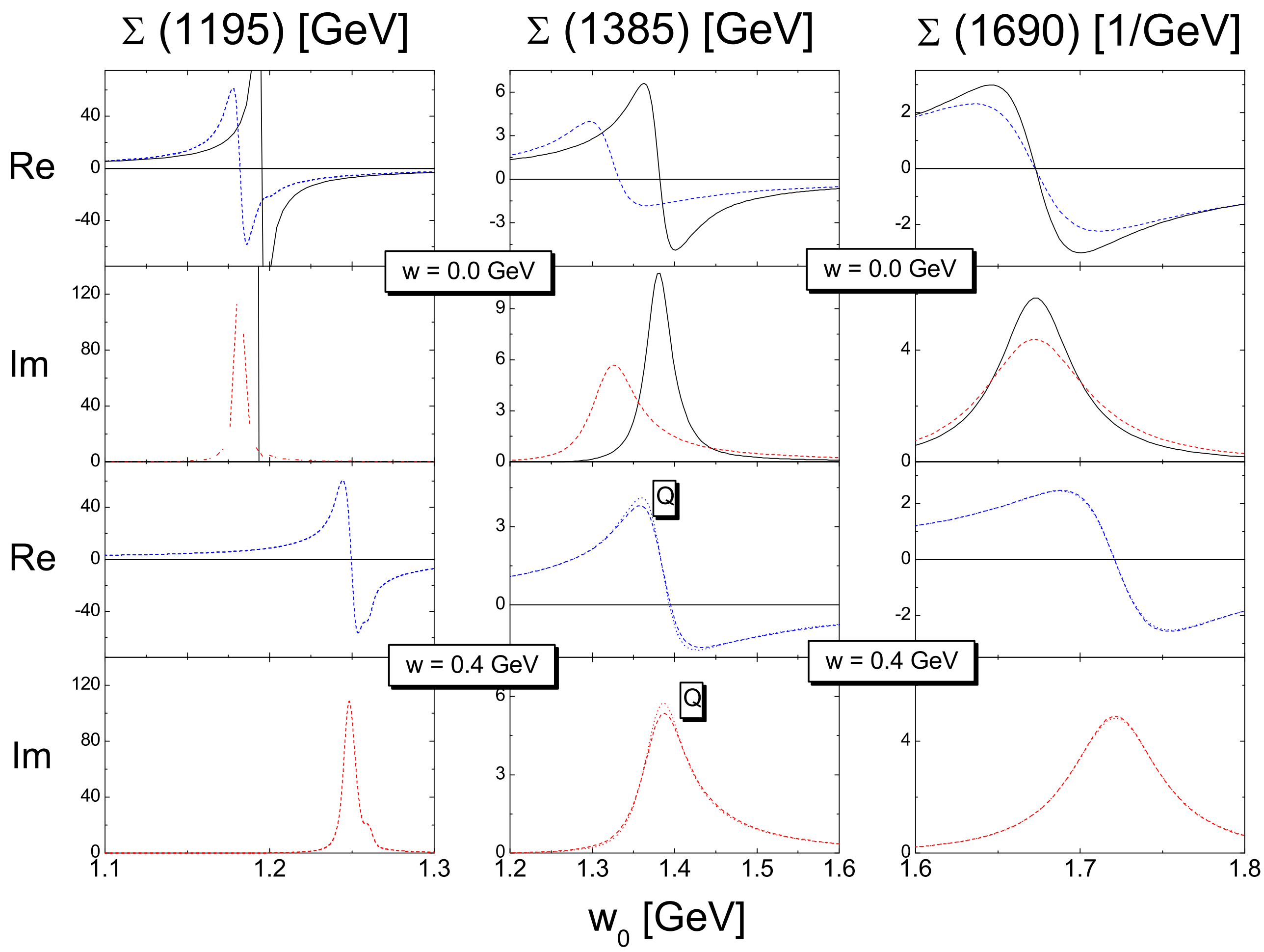
3. Numerical results: kaons and hyperons in the medium

When calculating the loop integrals we neglect the antikaon self energy in the vicinity of the negative-energy pole of the propagator

Reason: kaons containing a strange antiquark are subject to not too strong repulsion in the nuclear medium



Reduced amplitudes corresponding to Λ hyperons in isospin-symmetric nuclear matter of saturation density ($\rho_0 = 0.17$ fm^{-3})



The Σ hyperons at saturation density.

Mixing of the partial waves in the spherically symmetric medium:

If $\vec{w} = 0$ there is no mixing (rotational symmetry still holds, but if $\vec{w} \neq 0$ all partial waves mix

In the latter case we have two types of mixing:

In the 4×4 matrixban all 4 partial waves (S, P_1, P_3, D_3) are present

in the 2×2 only P_3 and D_3

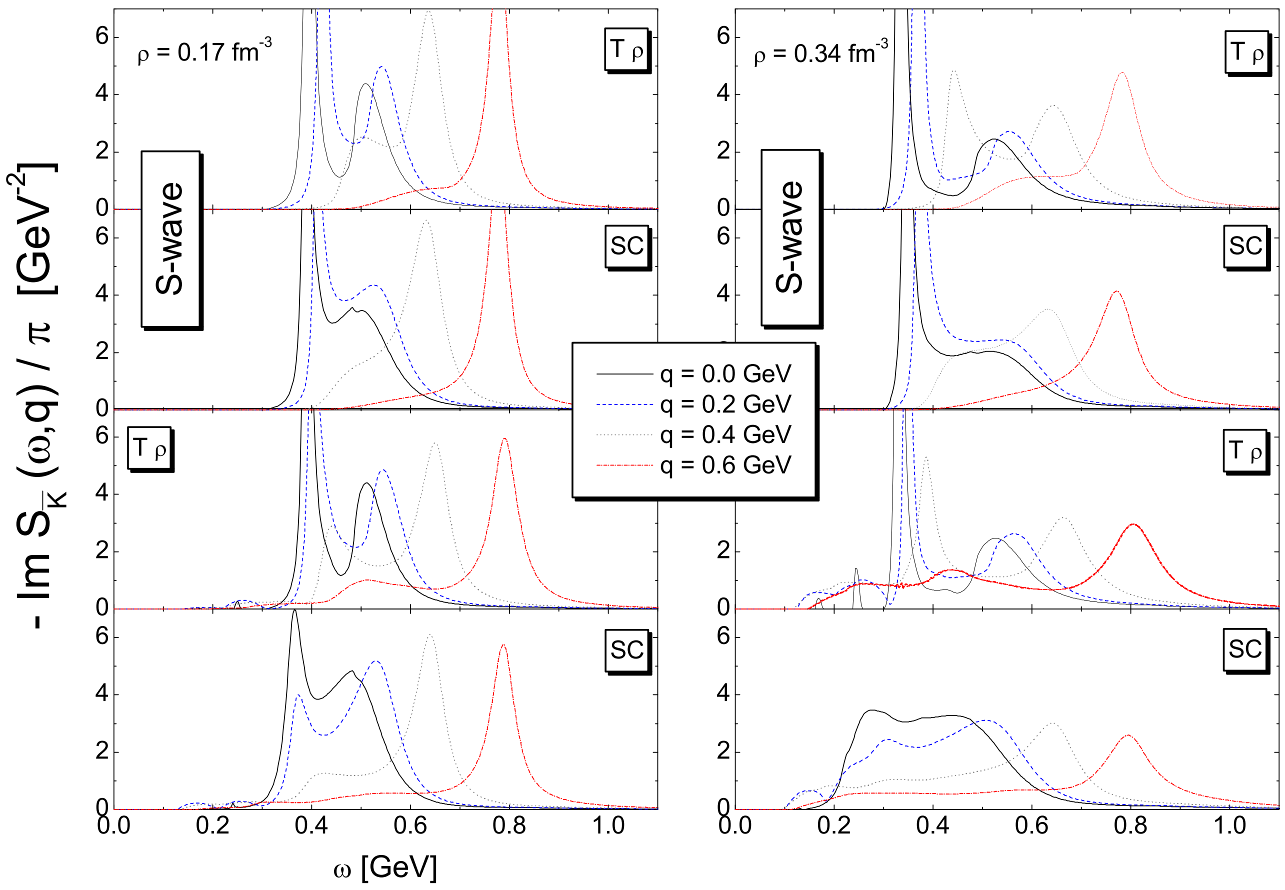
Explanation:

Which (rotational) quantum number remains good in the medium?

Answer: The helicity

There is separate mixing of states with helicity $\pm 1/2$, and those with helicity $\pm 3/2$

What happens to antikaons?



Spectral function of antikaons at saturation density ($\rho = 0.17$ fm^{-3}) and twice the saturation density ($\rho = 0.34$ fm^{-3}), in isospin symmetric medium

4. Pion-nucleon scattering and the pion in the nuclear medium

Above threshold $\sqrt{s} > m_N + m_\pi$ partial-wave amplitudes can be expressed through phase shifts $\delta_J^{(l)}(s)$ and inelasticity parameters $\eta_J^{(l)}(s)$:

$$t^{(\pm)}_{J-1/2}(s) = -i\,\frac{8\,\pi}{p_{\pi N}^{2J}}\,\frac{\eta^{(l)}_{J=l\pm\frac{1}{2}}(s)\,(e^{2\,i\,\delta^{(l)}_{J=l\pm\frac{1}{2}}(s)}-1)}{s+m_N^2-m_\pi^2\pm 2\,m_N\,\sqrt{s}}$$

For $\sqrt{s} < m_N - m_\pi$ use crossing

In between use analytic continuation, for example by the dispersion integral:

$$t_n^{(\pm)}(s) = d_n^{(\pm)}(s) + \int_{m_N+m_\pi}^{\infty}\frac{d\,w}{\pi}\,\frac{\Im t_n^{(\pm)}(w)}{w-\sqrt{s}-i\,\epsilon},$$

where $d_n^{(\pm)}(s)$ represents all left-hand cut contributions (which are dominated by s- and u-channel nucleon exchange)

However, there is a problem with off-shell pions and u-channel contributions!

Reason: The subthreshold extrapolation of the amplitudes is performed for on-shell pions, i.e.

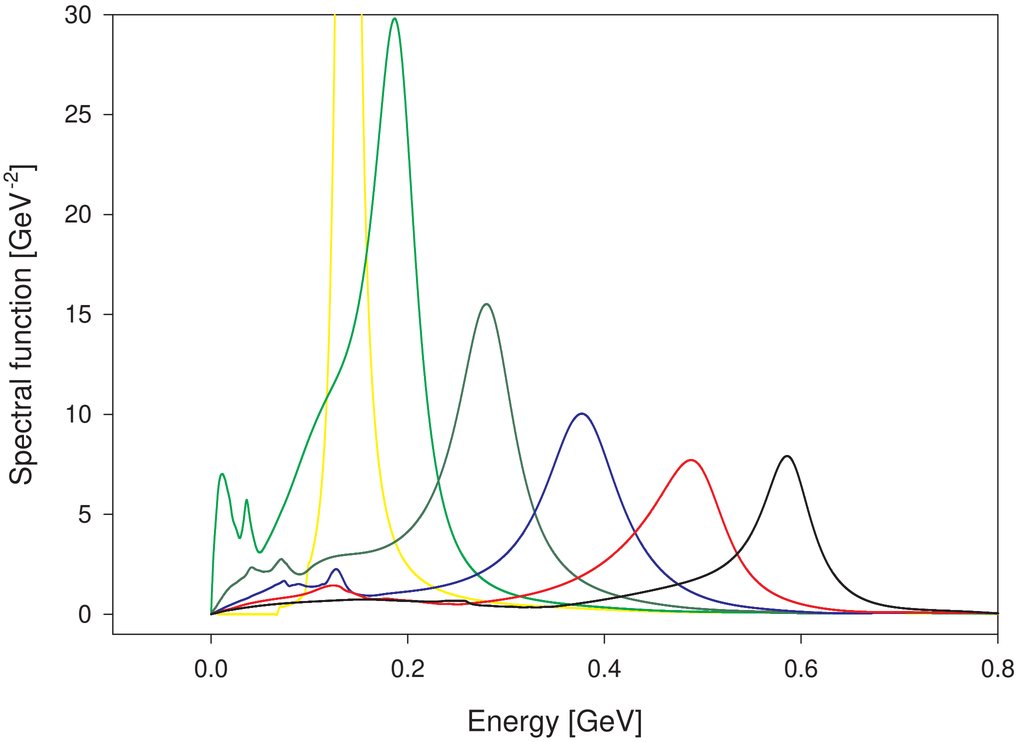
$$u = 2m_N^2 + 2m_\pi^2 - s \quad \text{(forward scattering)}$$

and not $u=(p-q)^2=m_N^2+q^2-2p\cdot q$, which applies for off-shell pions and is used in the diagrammatic evaluation

Result: large positive contributions to the imaginary part of the pion self energy from nucleon u-exchange, while diagrammatic evaluation gives real self energy

Cure: use amplitudes which depend not only on $s\equiv(p+q)^2$, but also on q^2.

Some preliminary results:



The N(1520) in the medium: