Analytic solutions of relativistic hydrodynamics *

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PLAN:

I. Relativistic hydrodynamics, intro

II. Bjorken's solution

III. Biro's solution

IV. New families of solutions

V. Summary and other topics

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I. Introduction

Motivation: Hydro description for central and non-central heavy ion collisions at RHIC

Scaling behaviour observed at CERN SPS and at RHIC

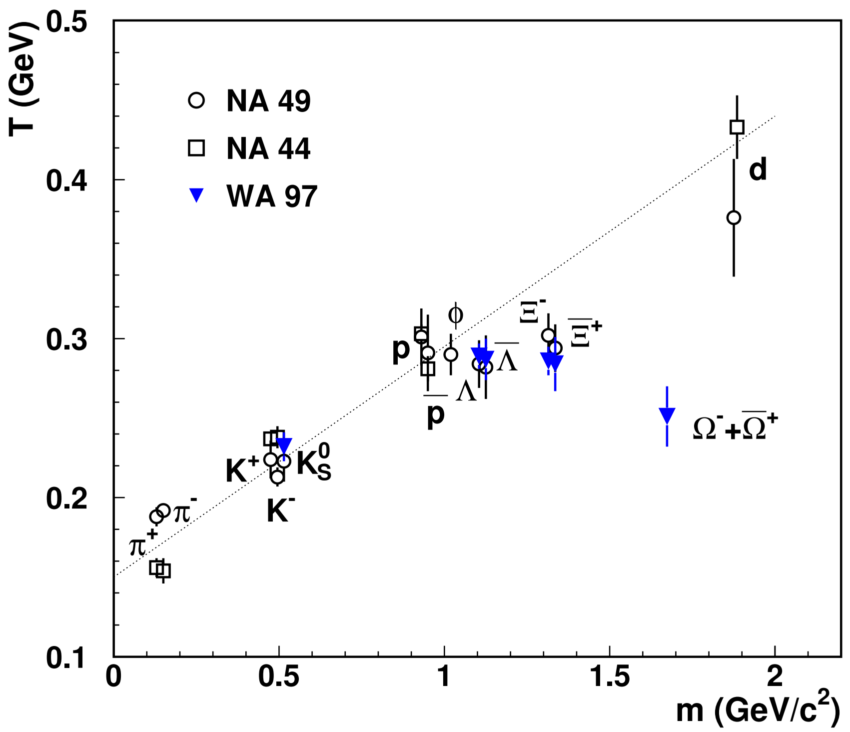
- collective behaviour $\rightarrow$ hydrodynamical description
- Scale invariance, Hubble-flow $\rightarrow$ self-similar hydro solutions
- simplicity but high energy $\rightarrow$ relativistic descriptions but with non-relativistic limiting solutions

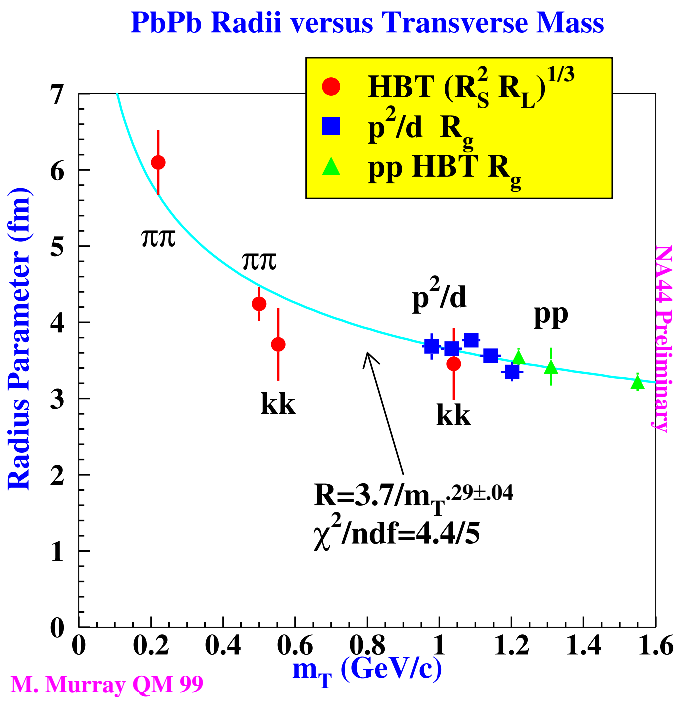
PRO's & Contra's

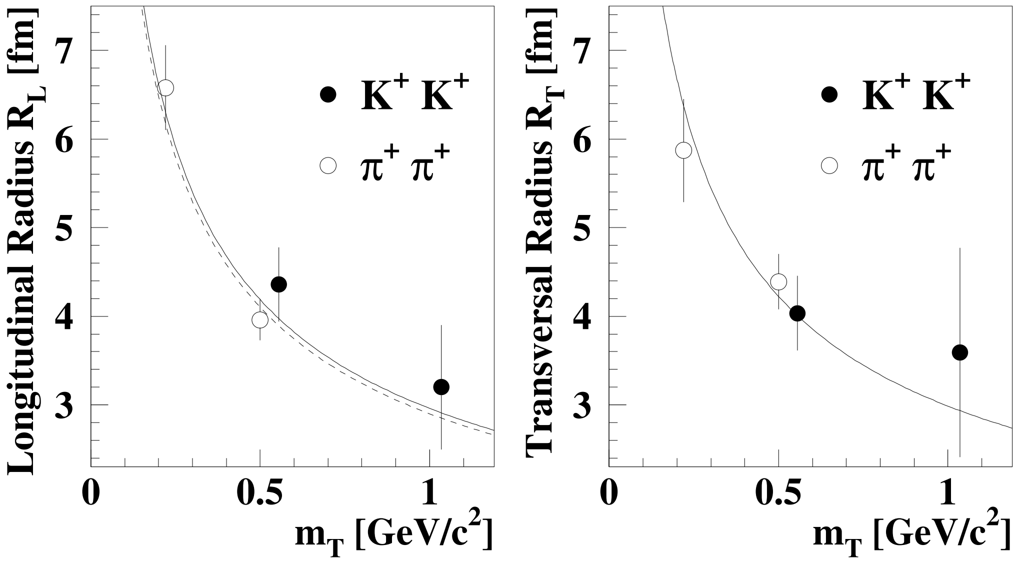
- Hydro can be applied within a wide range of phenomena, universality and scale invariance
- hydro description questionable for very small systems (a few 100 particles only)
- data are well described within a hydro approach, it is likely that hydro is valid for the fields (continuum)

Historical remarks

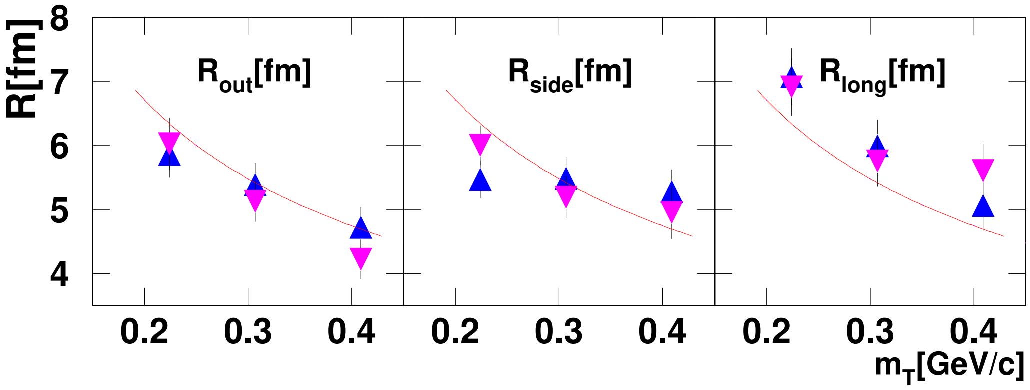
- Successfull hydrodynamical approach to high energy physics Landau, Bjorken
- Recent experimental results: scaling laws, scale invariance


T (GeV)
0.5
0.4
0.3
0.2
0.1
NA 49
NA 44
WA 97
d
Ξ^-
$\overline{\Xi}^+$
p
$\overline{\Lambda}$
Λ
$\overline{p}$
$\Omega^- + \overline{\Omega}^+$
K^+
K^0_S
K^-
π^-
π^+
0
0.5
1
1.5
2
m (GeV/c^2)


PbPb Radii versus Transverse Mass
HBT $(R_S^2 R_L)^{1/3}$
p^2/d R_g
pp HBT R_g
Radius Parameter (fm)
$\pi\pi$
$\pi\pi$
kk
p^2/d
pp
kk
NA44 Preliminary
$R=3.7/m_T^{.29\pm.04}$
$\chi^2/ndf=4.4/5$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
m_T (GeV/c)
M. Murray QM 99



$3/\sqrt{m_t}$ scaling at CERN SPS (NA44, Phys. Rev. Lett. 87 (2001) 112301)



$3/\sqrt{m_t}$ scaling at RHIC (T. Sugitate, hep-ph/0111460)

Relativistic hydrodynamical system of equations

$$\partial_\mu n u^\mu = 0, \quad (1)$$
$$\partial_\nu T^{\mu\nu} = 0. \quad (2)$$

$n \equiv n(x)$: density of a conserved charge,

$u^\mu \equiv u^\mu(x) = \gamma(1, \mathbf{v})$: four-velocity

normalized to $u^\mu u_\mu = \gamma^2(1 - \mathbf{v}^2) = 1$,

the energy-momentum tensor is $T^{\mu\nu}$.

Perfect fluid,

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p g^{\mu\nu}, \quad (3)$$

ϵ : relativistic energy density

p: pressure.

Ideal gas with contains massive conserved quanta,

$$\epsilon = mn + \kappa p, \quad (4)$$
$$p = nT. \quad (5)$$

Bjorken: $m = 0$, $\kappa = c_s^2$.

Softest point: $c_s^2 = 0$.

The energy-momentum conservation projected into parallel to u^μ and orthogonal to u^μ.

The relativistic energy and Euler equations:

$$u^\mu \partial_\mu \epsilon + (\epsilon + p)\partial_\mu u^\mu = 0, \quad (6)$$

$$u_\nu u^\mu \partial_\mu p + (\epsilon + p) u^\mu \partial_\mu u_\nu - \partial_\nu p = 0. \quad (7)$$

The energy equation can be rewritten as

$$u^\mu \partial_\mu T + \frac{1}{\kappa} T \partial_\mu u^\mu = 0. \quad (8)$$

Various symmetry classes are solved:

- 1+1 dimensional expansions
- 1+3 dimensional expansions :
- spherical
- cylindrical
- ellipsoidal

1+1 dimensional solutions

$x^\mu = (t, r_z)$, $x_\mu = (t, -r_z)$

$g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1, -1)$.

3 independent equations for

3 variables, n, T and v_z (equation of state!).

Self-similar solution.

A scale in the z direction, the scaling variable

$$x = \frac{r_z^2}{Z^2}, \tag{9}$$

Hubble type of self-similar longitudinal expansion,

$$v_z(t, r_z) = \frac{\dot{Z}(t)}{Z(t)} r_z, \tag{10}$$

In a relativistic notation,

$$\begin{aligned} u^\mu &= (\cosh\zeta, \sinh\zeta), && (11) \\ \tanh\zeta &= \frac{\dot{Z}(t)}{Z(t)} r_z, && (12) \\ \cosh\zeta &= \frac{1}{\sqrt{1 - \dot{Z}^2 x}} \equiv \gamma. && (13) \end{aligned}$$

New family of 1+1 d solutions

New variables:

$$\begin{aligned} \tau &= \sqrt{t^2 - r_z^2}, && (14) \\ \eta &= 0.5 \log \left(\frac{t + r_z}{t - r_z} \right). && (15) \end{aligned}$$

We find a new family of exact solutions

$$\begin{aligned} v_z &= \frac{r_z}{t} = \tanh(\eta), && (16) \\ x &= \frac{r_z^2}{\dot{Z}^2 t^2} = \frac{\tanh^2 \eta}{\dot{Z}^2}, && (17) \\ n(t, r_z) &= n_0 \frac{t_0}{\tau} \mathcal{V}(x), && (18) \\ p(t, r_z) &= p_0 \left(\frac{t_0}{\tau} \right)^{1+1/\kappa}, && (19) \\ T(t, r_z) &= T_0 \left(\frac{t_0}{\tau} \right)^{1/\kappa} \frac{1}{\mathcal{V}(x)}, && (20) \end{aligned}$$

where $p_0 = n_0 T_0$.

a new hydrodynamical solution is assigned to each non-negative function $\mathcal{T}(x)$.

valid also for massive particles.

depends parameterically on κ, (EOS).

Analyzis of the solution.

In all these solutions: $u^\mu = (\cosh\eta, 0, 0, \sinh\eta)$.

Bjorken's case:

$\dot{Z} = 1$ and $\mathcal{V}(x) = 1$.

The pressure, temperature and density depend only on τ.

The general case,

a scale, $\dot{Z}$, and an arbitrary scaling function $\mathcal{V}(x)$.

Physical meaning: non-flat rapidity distribution.

Single-particle spectra

$$S(x,\mathbf{k}) = C_\eta m_t \cosh(\eta - y) n \exp(-k^\mu u_\mu / T)\delta(\tau - \tau_f), \tag{21}$$

The single particle spectrum is

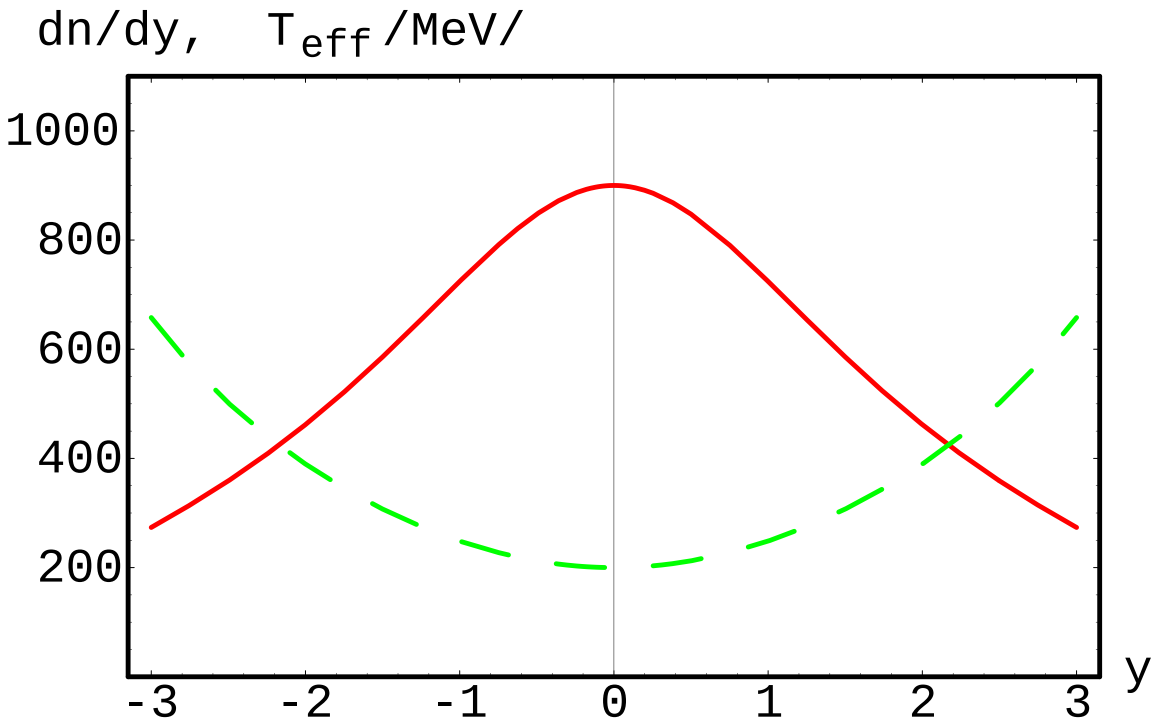
$$E\frac{d^3n}{d\mathbf{k}} \propto \int d\tau \tau d\eta S(x,\mathbf{k}). \tag{22}$$

$$\begin{aligned} E\frac{d^3n}{d\mathbf{k}} &= C' m_t \mathcal{V}\left(\frac{\tanh^2 y}{\dot{Z}_0^2}\right) \exp\left[-\frac{m_t}{T_{\text{eff}}(y)}\right], & (23)\\ \frac{dn}{dy} &= n_f \mathcal{V}\left(\frac{\tanh^2 y}{\dot{Z}_0^2}\right), & (24)\\ T_{\text{eff}}(y) &= \frac{T_f}{\mathcal{V}\left(\frac{\tanh^2 y}{\dot{Z}^2}\right)}, & (25) \end{aligned}$$

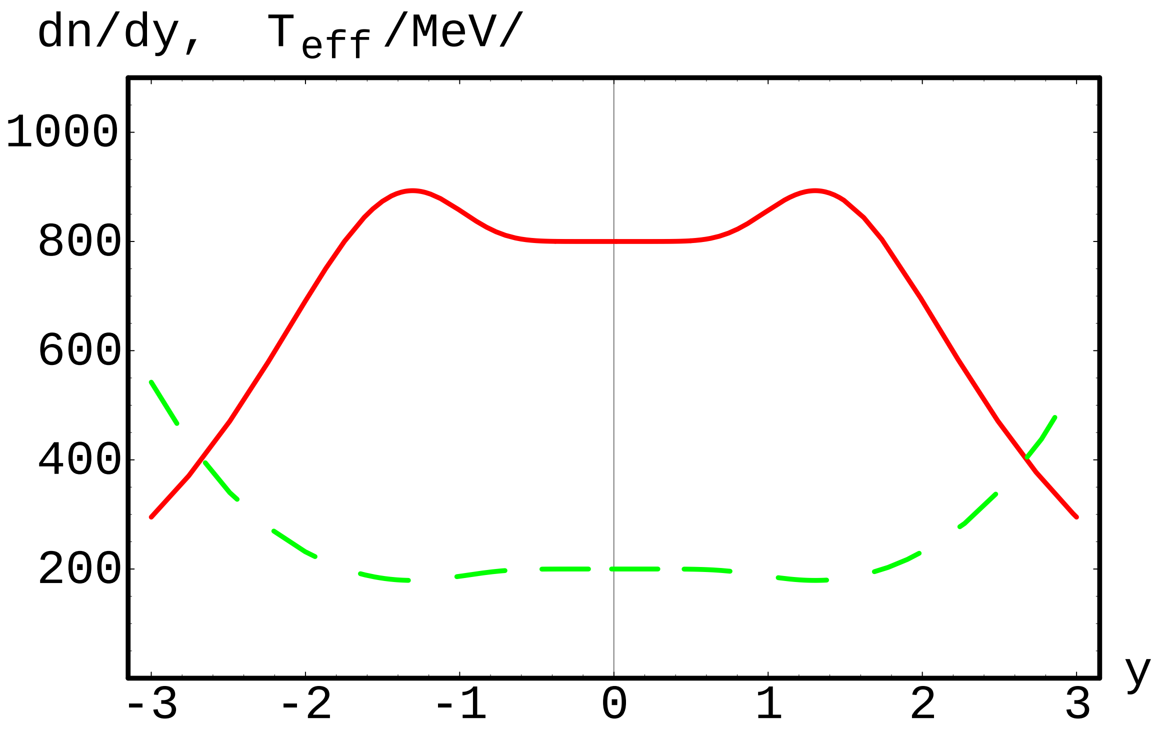
Bonus: $\mathcal{V}$ is a free fit function of the measurable rapidity distribution.

Price: A rapidity dependent effective temperature distribution

$$T_{\text{eff}}(y) = T_f \frac{dn/dy_0}{dn/d_y}, \tag{26}$$



$\mathcal{V}(x) = (1-x)^{(1/4)}$, $\dot{Z} = \tanh(4)$, $n_f = 900$ and $T_f = 200$ MeV.



$\mathcal{V}(x) = \sqrt{1 + 1.6x^4 - 2.6x^8}$, using $\dot{Z}_0^2 = 1$, $n_f = 800$ and $T_f = 200$ MeV.

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Summary and discussion

- New family of relativistic hydrodynamical solutions were found
- Generalization of Bjorken's solution for finite dn/dy
- Price: rapidity dependent effective temperature
- Generalizations for 1+3 dimensional expansions are done
- Elliptic and cylindrical families
- Well-defined non-relativistic limits and non-rel solutions
- Related to the Buda-Lund hydro parameterization (Fits data, Ster)