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Finite temperature spectral function of the σ meson from large N expansion

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Abstract. The spectral function of the scalar-isoscalar channel of the $O(N)$ symmetric linear σ model is studied in the broken symmetry phase. The investigation is based on the leading order evaluation of the self-energy in the limit of large number of Goldstone bosons. We describe its temperature dependent variation in the whole low temperature phase. This variation closely reflects the trajectory of the scalar-isoscalar quasiparticle pole. In the model with no explicit chiral symmetry breaking we have studied near the critical point also the corresponding dynamical exponent.

Keywords: spectral function, linear σ model, large N approximation
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1. Introduction

The nature of the scalar meson sector of QCD, with particular emphasis on the broad σ meson receives increasing attention [1,2]. This sector describes the dynamics of the order parameter of chiral symmetry, which is the fundamental symmetry of quantum chromodynamics, realized in the broken symmetry phase. Therefore the response of this multiplet to the variation of the temperature or the baryonic density are directly related to the (partial) restoration of the chiral symmetry.

We concentrate our study on the σ-meson, therefore the $O(4)$ symmetric linear σ-model provides an adequate field theoretical framework. The finite temperature/baryon density behaviour of the σ meson has been studied extensively also with help of this model in the last 15 years [3]. Since in any realistic parametrisation the model is strongly (self)-coupled, there is no basis for the use of the simplest

perturbative techniques.

Improved (resummed) finite temperature perturbation theory was applied to the thermodynamics of the model [4] and led to the conclusion that σ becomes a narrow resonance with the increase of the temperature when its mass approaches the (temperature dependent) two-pion threshold. The narrowing is due to the smaller phase space available for the dominant $\sigma \to 2\pi$ decay. This situation manifests itself in high energy heavy ion collisions by enhanced two-pion production near this threshold and the appearance of a high intensity narrow peak in the two-γ spectra superposed on the broad background arising from the π^0 decay.

One might remark, that results of the simplest perturbative treatments turn out to be sensitive to the details of the renormalisation prescription, do not reflect the so-called hybridisation phenomenon occuring between the σ and some composite channels (see below), and finally, do not account correctly for the continuous nature of the chiral symmetry restoration.

There was some notable progress in the non-perturbative lattice determination of the QCD spectral functions, which did not provide to date competitive results in the scalar-isoscalar channel [5, 6].

Our investigation relies on the large N treatment of the thermodynamics of the linear sigma model, where N refers to the number of the Goldstone bosons. The leading order approximation can be shown to avoid all problems of the usual perturbative treatment listed above. Additional credit for the use of large N techniques is provided by its very successful application to dynamical critical phenomena occuring in superfluid helium [7–9] and by some very promising first attempts to go beyond Hartree-type approximations in exploring the mechanism of thermalisation for quantum fields [10].

A more detailed report of our results obtained in the model with no explicit $O(4)$ symmetry breaking can be found in Ref.[11] and a detailed investigation of the most realistically parametrised system with explicit symmetry breaking is presently near completion. Here we describe the more transparent (though phenomenologically less appealing) case when no explicit symmetry breaking is applied.

2. Leading order large N calculation of the propagators

The Lagrangian of the N-component scalar field theoretical model is parametrised for the large N expansion in the following way:

$$L = \frac{1}{2}[\partial_\mu \phi^a \partial^\mu \phi^a - m^2 \phi^a \phi^a] - \frac{\lambda}{24N}(\phi^a)^2(\phi^b)^2. \tag{1}$$

The direction of the symmetry breaking is chosen along the $a = 1$ direction and is expressed by the shift in the corresponding field component:

$$\phi^a \to (\sqrt{N}\Phi + \varphi^1, \varphi^i), \qquad i = 2, ..., N. \tag{2}$$

It is worth to note that in this parametrisation Φ is related to the the phenomenological f_π parameter at $N = 4$ by $f_\pi = 2\Phi$.

The equation of state is determined by the vanishing of the coefficient of φ^1 in the shifted Lagrangian:

$$\langle\frac{\delta L}{\delta\varphi^1}\rangle = 0 = \sqrt{N}\Phi\Big[m^2 + \frac{\lambda}{6}\Phi^2 + \frac{\lambda}{6N}\langle(\varphi^a)^2\rangle\Big]. \tag{3}$$

To leading order in N the propagator of the transversal (Goldstone) modes receives the same tadpole contribution:

$$G^{-1}_{Goldstone}(p) = p^2 - m^2 - \frac{\lambda}{6}\Phi^2 - \frac{\lambda}{6N}\langle(\varphi^a)^2\rangle, \tag{4}$$

which in view of (3) leads to the masslessness of the Goldstone modes.

The equation of state should be renormalised due to the ultraviolet divergence of $(\lambda/N)\langle(\varphi^a)^2\rangle$. To leading order in N one retains only the contribution from the massless Goldstone modes to the field fluctuations. The equation of state is rewritten with the renormalised couplings defined through the relations

$$\frac{m^2}{\lambda} + \frac{\Lambda^2}{96\pi^2} = \frac{m_R^2}{\lambda_R}, \qquad \frac{1}{\lambda} + \frac{1}{96\pi^2}\ln\frac{e\Lambda^2}{M_0^2} = \frac{1}{\lambda_R}, \tag{5}$$

as

$$m_R^2 + \frac{\lambda_R}{6}\Phi(T)^2 + \frac{\lambda_R}{72}T^2 = 0. \tag{6}$$

This expression can be rewritten as an expression for the temperature variation of the order parameter relative to its $T = 0$ value:

$$\Phi^2(T) = \Phi_0^2 - \frac{1}{12}T^2, \tag{7}$$

which yields for the critical temperature the prediction: $T_c^2 = 12\Phi_0^2 = 3f_\pi^2 \approx (161\mathrm{MeV})^2$. This is in very good agreement with the critical temperature determined with lattice simulations [12].

The propagator of the σ field receives to this order additional contribution from the infinite chain of bubble diagrams:

$$G_\sigma^{-1} = p^2 - \frac{\lambda}{3}\Phi^2(T)\frac{1}{1-\lambda b(p)/6}, \tag{8}$$

where $b(p)$ stands for the analytic expression of a single bubble. After introducing the renormalised coupling, the renormalised, $T = 0$ expression of the bubble is given by:

$$b_0(p) = \frac{1}{16\pi^2}\ln\frac{-p^2}{M_0^2}, \tag{9}$$

where M_0 denotes the renormalisation scale. The $T = 0$ σ-propagator looks like

$$G_\sigma^{-1}(p) = p^2 - \frac{\lambda_R}{3}\Phi_0^2\frac{1}{1-\lambda_R\ln(-p^2/M_0^2)/96\pi^2}. \tag{10}$$

In Fig.1 we show the position of the $T = 0$ complex σ-pole, determined from the equation $G_\sigma^{-1} = 0$, as a function of λ_R in units of f_π. This pole is a physical resonance, since it has negative imaginary part. It is obvious from the figure that its real part cannot exceed $\sim 3.5 f_\pi$, while the imaginary part gradually increases with the growth of λ_R. Choosing $\lambda_R = 310$ we find $M_\sigma/\Gamma_\sigma \sim 1$, which is close to the phenomenologically observed value, though the mass itself is too small.

A pure imaginary pole ($p_0 = iM_L$) of the σ-propagator is found for every given T. The theory is meaningful only for momentum scales which lie considerably below the tachyonic frequency. This limitates the allowed λ_R range from above. In Fig.1 we also display the ratio M_L/M_σ for several temperatures. The T-dependence is not very important therefore the limitation in the choice of λ_R is nearly independent of T.

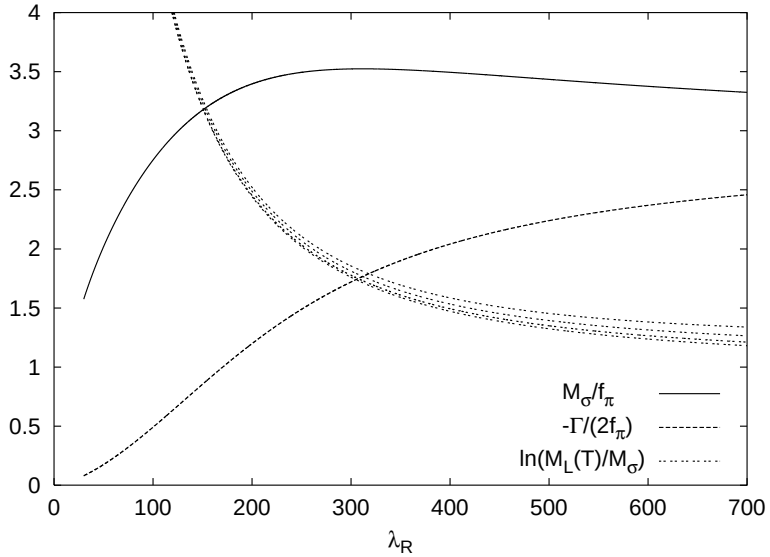

Fig. 1. The real and imaginary part of the $T = 0$ pole mass of the σ particle as a function of λ_R. We also display the location of the pure imaginary tachyonic pole, M_L.

3. The scalar-isoscalar spectral function

At finite temperature an additional term appears in the expression of $b(p)$, which is defined in the upper halfplane of the complex p_0 variable:

$$b_T^{>}(p_0) = \frac{1}{8\pi^2}\int_0^\infty dx \frac{1}{e^x - 1}\Big[\frac{1}{z-x} - \frac{1}{z+x}\Big], \qquad z = \frac{p_0}{2T}, \qquad \mathrm{Im}\ p_0 > 0. \quad (11)$$

One can compute the real and imaginary parts of the complete expression of $b(p) = b_0(p) + b_T(p)$ when $\mathrm{Im} p_0 \to +0$. It determines the spectral function in the scalar-

isoscalar channel:

$$\rho(p_0, \mathbf{p}) = -\frac{1}{\pi}\mathrm{Im}G_\sigma(p_0 + i0, \mathbf{p}). \tag{12}$$

Similarly to G_σ one can evaluate the propagator of the composite field $\phi^a\phi^a - \langle\phi^a\rangle^2$:

$$F(p) = \frac{p^2 b(p)/6 + \Phi^2(T)/3}{p^2(1 - \lambda_R b(p)/6) - \lambda_R \Phi^2(T)/3}. \tag{13}$$

Obviously, its denominator is the same as that of G_σ, which reflects the phenomenon of hybridisation.

In the two parts of Fig.2 we give the spectral functions of G_σ and of F for $\mathbf{p} = 0$ as functions of p_0 for various temperatures. One observes that both $T = 0$ curves are peaked at p_0 values which agree with the real part of the σ-pole. The question we addressed was: can one interpret the shift of the maxima for increasing temperatures as a reflection of the finite temperature shift of the quasi-particle pole?

In order to answer this question one has to continue the propagators analytically into the lower p_0 halfplane (onto the second Riemann sheet) and find the poles smoothly joining the location of the $T = 0$ pole. The expression of the zero temperature piece $b_0(p)$ is continued in a unique manner into this region. The harder task is the continuation of the finite temperature contribution. One can check that if one chooses $b_T(p_0)$ to be analytic on the positive real axis, then the relation between $b_T^>$ and its continuation into the lower halfplane $b_T^<$ is the following:

$$b_T^<(p_0) = b_T^>(p_0) - \frac{i}{4\pi}\frac{1}{\exp(p_0/2T) - 1}, \qquad \mathrm{Im}\ p_0 < 0, \qquad \mathrm{Re}\ p_0 > 0. \tag{14}$$

With help of this expression of $b(p_0)$ valid on the second Riemann sheet one can find for every T the point where $G_\sigma(p_0)^{-1}$ or $F(p_0)^{-1}$ vanishes.

In Fig.3 we display the imaginary part of the trajectory of the σ-pole as a function of the temperature. Its real part gradually decreases with increasing temperature. The imaginary part slightly increases at the same time. At some $T = T_{imag}$ the pole position reaches the negative portion of the imaginary axis. At the same temperature a "mirror" pole touches the same point of the imaginary axis arriving from the left. The "collision" of the two poles leads to the birth of two purely imaginary poles, present for higher temperatures. One of them moves towards, the other one away from the origin along the imaginary axis. The narrowing of the σ quasi-particle starts only for temperatures $T > T_{imag}$ and the same is true for the start of the "threshold enhancement" of the spectral function.

The $O(N)$ symmetry is restored at the temperature for which the pole moving in this direction reaches the origin. In the temperature range when the poles are found on the negative imaginary axis, the spectral functions become cuspy. The closer moves the pole along the imaginary axis to the origin, the narrower the shape of the spectral function becomes. Finally for the temperature when one of the poles arrives to the origin one finds analytically that

$$\rho(p_0, T \sim T_c) \sim \delta(p_0)/p_0, \tag{15}$$

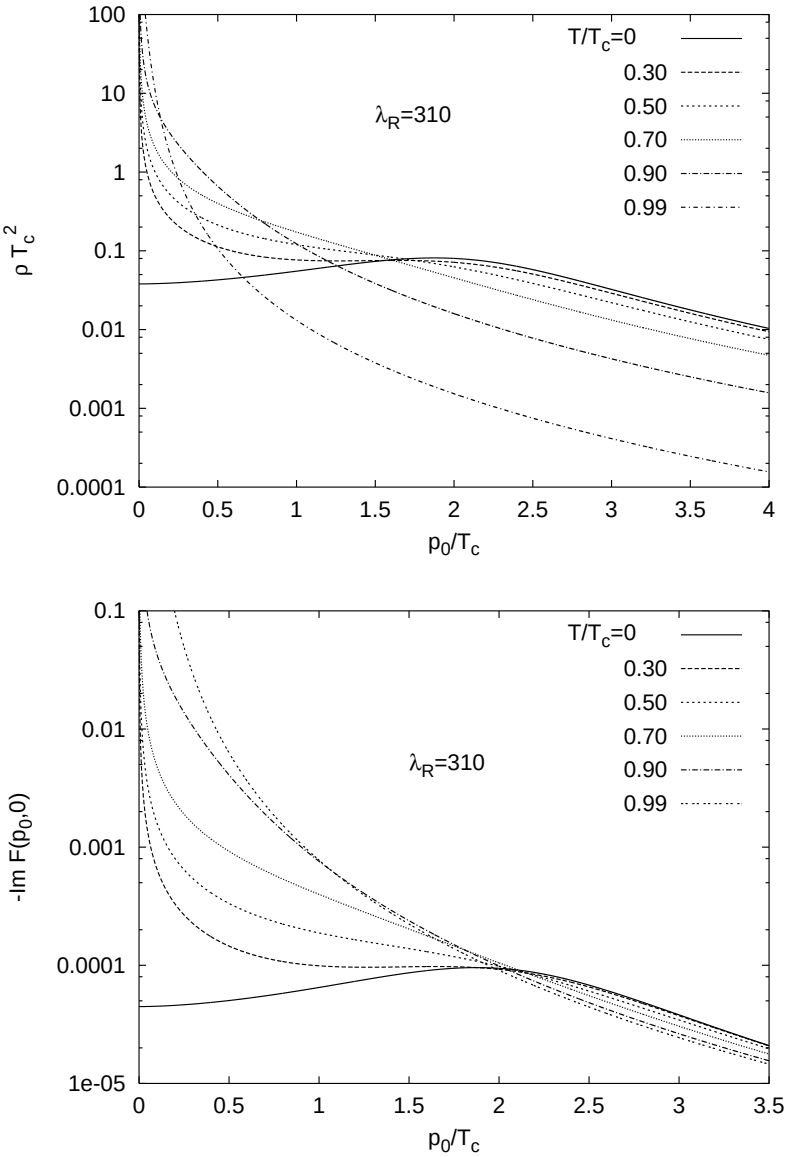

Fig. 2. The spectral function of the elementary and of the quadratic composite scalar-isoscalar field as a function of the frequency p_0 for different temperatures

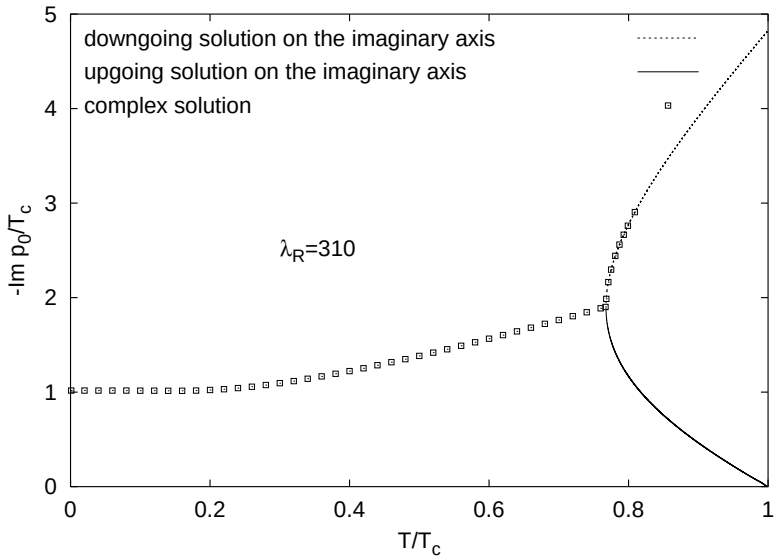

Fig. 3. The imaginary part of the σ-pole in the $4^{\rm th}$ quadrant of the complex p_0-plane including the negative imaginary axis. Depending on the details of the numerical algorithm used, the complex solution may continue into the up-going solution as well

where the δ-function singularity corresponds to the maximal threshold enhancement.

From the point of view of the spectral function the whole temperature range $T_{imag} < T < T_c$ can be called the threshold enhancement regime, since in this temperature interval the maximal signal comes from the direct neighbourhood of the imaginary axis. We believe that this is the generic behaviour, since there is no reason to expect that from every starting point in the p_0-plane the pole "flows" directly to the origin, without hitting the imaginary axis.

4. Critical region

In the vicinity of the critical point, where $\xi = T/8\pi\Phi^2(T)$ is the dominant length scale and the condition $p_0, |\mathbf{p}| << T$ is also fulfilled, one can derive an equation also for the soft modes with nonzero momentum:

$$-3i|\mathbf{p}|\xi\frac{p_0^2-\mathbf{p}^2}{|\mathbf{p}|^2}\ln\frac{p_0-|\mathbf{p}|}{p_0+|\mathbf{p}|}-\frac{1}{4\pi}\frac{(p_0^2-\mathbf{p}^2)\xi}{T_c}\ln\frac{p_0^2-\mathbf{p}^2}{T_c^2}=1. \qquad (16)$$

Its solution in the approximation, when on the left hand side only the first term is retained exhibits the form of dynamical scaling [7,8]: $p_0 = |\mathbf{p}|^{\tilde{z}} f(|\mathbf{p}|\xi)$ with $\tilde{z} = 1$.

The second term provides the leading correction to scaling. In $O(N)$ models, the dynamical exponent $z = d/2$ has been obtained for finite N on the basis of scaling and renormalisation group arguments, where in our case $d = 3$ [13–15]. For the correct interpretation of the situation it is important that there are two distinct hydrodynamical regions in the $O(N)$ model for large N [13]. In the true critical region $z = d/2$ is valid, in a precritical region $\tilde{z} = 1 - 8S_d/Nd + \mathcal{O}(1/N^2)$, where $S_d = 2/\pi^2$ for $d = 3$. The first region shrinks when N becomes large and completely disappears at $N = \infty$.

5. Conclusion

The aim of this talk was to clarify the relationship of the temperature dependence of the spectral function characterizing the excitations of the chiral order parameter and the quasi-particle pole with the same quantum numbers (the σ-meson). The application of the leading order analysis in a large N approximation, where N is the number of Goldstone modes, was shown to avoid most of the conceptual problems related to the application of conventional perturbative approaches. The pole of the σ-propagator continued analytically into the lower p_0 halfplane follows a trajectory, which describes satisfactorily the behaviour of the spectral function both near $T = 0$ and in the critical region. The transition between the two regimes happens around T_{imag}, the temperature when the pole becomes purely imaginary.

The line of analysis presented here has been applied also to the model with explicit $O(4)$-breaking external field [11, 16], with parameters chosen the closest possible to the measured characteristics of the $\sigma - \pi$ meson system. The behaviour found in this analysis proved generic also for the system with finite baryonic charge density.

In view of the clear physical picture we found for the mechanism behind the threshold enhancement of the scalar-isoscalar spectral function, it is of interest to compute the next-to-leading corrections. This will enable us to see to what extent can one consider the present approach also quantitatively relevant.

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Notes

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