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Broad resonances in transport theory

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Abstract. We discuss the extension of the transport theoretical framework to include states with a broad mass distribution. We focus on the proper life time and cross sections for a state with an arbitrarily given invariant mass.

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1. Introduction

Boltzmann type equations have turned out to be extremely successful in the description of heavy-ion reactions over a broad range of energies [1]. For low energies the colliding states can be regarded as asymptotically stable. For the description of such *on-shell* states Boltzmann type equations have the generic form

$$\frac{d}{dt}f(t,\vec{x},\vec{p}) \equiv \left(\partial_t + \frac{\vec{p}}{E}\vec{\nabla}_x - \vec{\nabla}_x U(t,\vec{x})\vec{\nabla}_p\right) f(t,\vec{x},\vec{p}) = I_{\rm coll} \tag{1}$$

where f denotes the phase space distribution. The corresponding energy E which appears in (1) in the so-called drift term is fixed by the on-shell relation $E = \sqrt{\vec{p}^2 + M^2}$. Note that f does not explicitly depend on E. In (1) U is the mean-field potential (appearing in the Vlasov term) while $I_{\rm coll}$ encodes all the collisions of the particle type under consideration. In practice, equations of type (1) are solved by a test particle ansatz

$$f(t,\vec{x},\vec{p}) \sim \sum_i \delta^{(3)}(\vec{x}-\vec{x}_i(t))\,\delta^{(3)}(\vec{p}-\vec{p}_i(t)) \tag{2}$$

where $\vec{x}_i(t)$ and $\vec{p}_i(t)$ denote position and momentum of test particle i at time t. Again, the energy of test particle i is not a free variable but given by the corresponding on-shell condition.

In spite of the fact that such equations can describe a large variety of experimental data extensions of this framework are important once higher energies are envisaged in heavy-ion collisions. First of all, in the collisions of the participating nucleons resonances might get excited, i.e. intermediate states with large decay widths. Second, even the spectral functions of states which are asymptotically stable in vacuum are influenced by collisional broadening. In the following we focus on the first aspect. Figs. 1 and 2 present examples for spectral functions of resonances. Fig. 1 shows the spectral function of a ρ meson for vacuum and nuclear matter density [2]. To be specific it is a ρ meson at rest with respect to the nuclear environment. Clearly already for the vacuum case the ρ meson is not well described by an asymptotic state, i.e. by a δ-function. In the nuclear medium additional broadening and even a new peak structure is caused by the coupling to resonance-hole states. Especially the additional peak shows the collective $N^*(1520)$-hole excitation (cf. also the contribution of G. Wolf [3]). Fig. 2 shows the broad spectral distri-

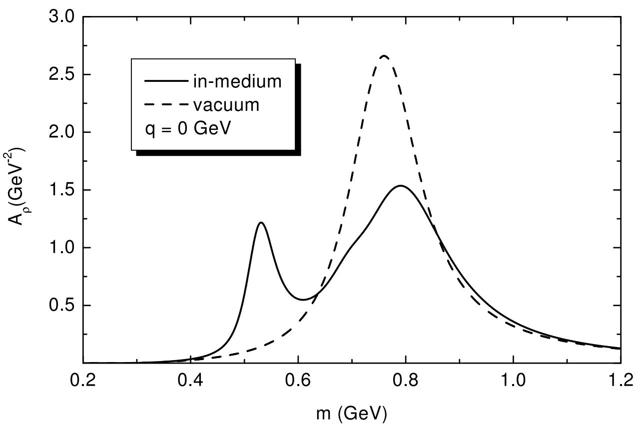

Fig. 1. The spectral function of a ρ meson for vacuum and for nuclear matter density [2].

bution of the $\Delta(1232)$ resonance for the vacuum case. In heavy-ion reactions the Δ is formed in collisions $NN \to N\Delta$. Due to its decay branch $\Delta \to N\pi$ it is an important source for the pion production process $NN \to NN\pi$. We will come back to that point below.

Once degrees of freedom with broad spectral functions become relevant they should be properly included in transport simulations. A principal remark is in order here: Resonances as unstable states do not show up in detectors but in scattering phase shifts of stable states. Therefore it is in principle conceivable to consider in transport simulations the propagation and scattering of stable states only and incorporate the resonances in the scattering cross sections. E.g. the previously mentioned Δ resonance shows up in the πN cross section. What is missing in such

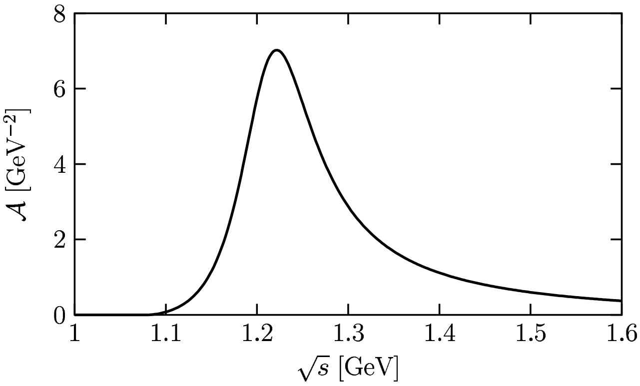

Fig. 2. The (vacuum) spectral function of the $\Delta(1232)$ [4].

an approach is the possibility that e.g. a Δ once formed in a πN collision travels through the medium before it decays into πN again. During its journey however it might appear that instead of decaying again the Δ gets destroyed by a nucleon on account of the reaction $\Delta N \to NN$. If we only look at the initial and final stable states such a multi-step process is nothing but a three-body reaction $\pi NN \to NN$. In practice, such reactions are hard to incorporate into transport simulations. This problem is however circumvented once resonances are introduced as explicit degrees of freedom which are allowed to propagate, scatter and decay. In this way, multi-body processes are split up into $2 \to 2$, $2 \to 1$ and $1 \to 2$ processes which are easier to handle. Therefore the key issue is to work out how resonances or, more general, states with a continuous mass distribution propagate, scatter and decay properly [4–6].

Clearly for such states (2) has to be generalized to

$$f(t,\vec{x},\vec{p},M^2) \sim \sum_i \delta^{(3)}(\vec{x}-\vec{x}_i(t))\,\delta^{(3)}(\vec{p}-\vec{p}_i(t))\,\delta(M^2-M_i^2(t)) \qquad (3)$$

where the test particle i now has an arbitrary mass M_i which even might change with time. For details on that point we refer to [4, 5] and references therein.

2. General considerations about life times of resonances

Once a resonance with mass M and momentum $\vec{p}$ is created in a collision we have to answer the question how long it will live. We expect a radiation law which yields the survival probability p at time t: $p(t) \sim e^{-t/\tau(M,\vec{p})}$ where the life time τ in general is a function of the invariant mass of the resonance and its momentum.

In the following we work in the rest system of the resonance and do not display the momentum dependence explicitly. What do we know about the life time τ? Typically one attributes a life time to a resonance species, e.g. the "life time" of the Δ resonance is $\langle\tau\rangle = 1/\Gamma = 1/(120 \text{ MeV})$ where Γ is the peak width of the Δ [7]. This quantity $\langle\tau\rangle$, however, is *not* the life time of a Δ with a specific invariant mass (not even the life time of a Δ with mass $M = 1232$ MeV) but it is the average life time of the resonance species Δ:

$$\langle\tau\rangle = \int \frac{dM}{2\pi}\, \tau(M)\, \mathcal{A}(M) \tag{4}$$

with the (here non-relativistic) spectral function $\mathcal{A}$. If the resonance is rather sharp (like e.g. the ω meson [7] but not the Δ), i.e. if one deals with a quasi-particle (quasi-asymptotic state) where the only relevant invariant mass is the peak mass $M_{\text{on-shell}}$ one finds

$$\tau(M_{\text{on-shell}}) = \langle\tau\rangle = 1/\Gamma(M_{\text{on-shell}})\,. \tag{5}$$

In contrast to the case of sharp resonances the proper definition of $\tau(M)$ is not a settled issue at present for broad resonances. In addition, if there is more than one decay mode we are interested in $\tau_{\text{partial}}(M)$ for every particular decay channel. In the following we shall discuss two different prescriptions for the life times. To get already a taste how different these two alternatives are one might have a look at Fig. 3. Both curves will be explained in the following.

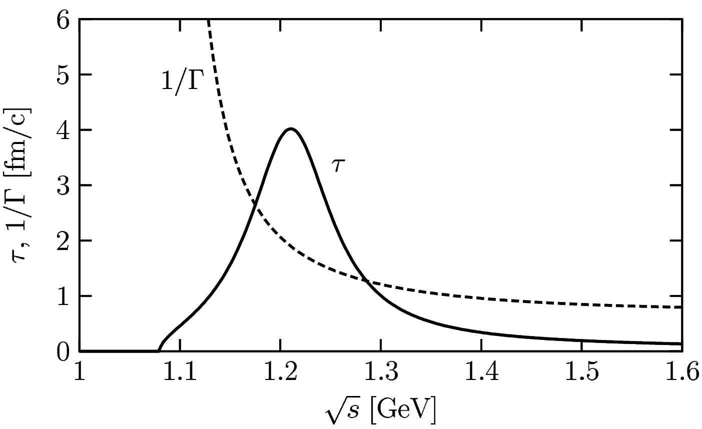

Fig. 3. Comparison of two different life time prescriptions [4]. See main text for details.

The standard recipe utilized in most of the transport codes is indeed to use the inverse decay width for the respective invariant mass. This means that for any invariant mass of the resonance one calculates the life time as if this invariant mass is the only accessible one, i.e. one uses (5) for arbitrary M:

$$\tau(M) \stackrel{?}{=} 1/\Gamma(M) \qquad\qquad \text{standard recipe.} \tag{6}$$

E.g. for the partial life time of a Δ of mass M with respect to the πN decay channel one uses the inverse of the (energy dependent) decay width $\Gamma_{\pi N}(\sqrt{s})$ for $\sqrt{s} = M$. For vacuum this yields the dashed curve in Fig. 3. Correspondingly, one can also attribute a partial life time to an absorption process: E.g. the scattering $\Delta N \to NN$ can be understood formally as a decay of the Δ into one nucleon hole and two nucleons. According to the standard recipe this absorption life time $\tau_{\rm abs}(M)$ is given by the inverse of the scattering width $\Gamma_{\Delta N \to NN}(M)$. In turn the scattering width is proportional to a cross section:

$$\Gamma_{\Delta N \to NN}(M) \sim \frac{d\sigma_{\Delta N \to NN}}{d\Omega}\,. \tag{7}$$

We have to stress here that this "cross section" appearing in (7) is *not* a measurable quantity. In principle it can be calculated according to Feynman's rules as if the Δ with the invariant mass M was an asymptotic state. In fact, we here define the quantity $d\sigma_{\Delta N \to NN}/d\Omega$ to be given in that way. For true decay processes one has to ask about the proper life time and correspondingly here the question is: What is the correct cross section for a scattering process with a resonance in an entrance channel? Indeed, using $d\sigma_{\Delta N \to NN}/d\Omega$ for the cross section follows the same philosophy as using $1/\Gamma$ for the life time: For each invariant mass one calculates the life time or cross section as if this was the only accessible mass. We will come back to that point below.

The alternative to the commonly used $1/\Gamma$ recipe for the life time is based on the following consideration: A resonance is an intermediate state formed in a scattering event. Therefore the life time of a resonance should be intimately connected to the scattering phase shift as the latter contains the information how long the scatterers remain in the resonant state. As shown in [8] the proper relation for vacuum and only one decay channel is given by

$$\tau_{\rm single\,dec} = \frac{\partial \delta}{\partial E} = \frac{1}{2}\mathcal{A}\,(1-K) \tag{8}$$

with the scattering phase shift

$$\tan\delta = \frac{\Gamma/2}{\frac{\vec{p}^2}{2m} + {\rm Re}\bar{\Sigma}^{\rm ret} - E}\,, \tag{9}$$

the spectral function

$$\mathcal{A} = \frac{\Gamma}{(E - \frac{\vec{p}^2}{2m} - {\rm Re}\bar{\Sigma}^{\rm ret})^2 + \frac{1}{4}\Gamma^2} \tag{10}$$

and

$$K = \frac{\partial \mathrm{Re}\bar{\Sigma}^{\mathrm{ret}}}{\partial E} + \frac{E - \frac{\vec{p}^2}{2m} - \mathrm{Re}\bar{\Sigma}^{\mathrm{ret}}}{\Gamma} \frac{\partial \Gamma}{\partial E} \,. \tag{11}$$

Here Γ is the decay width, i.e. the imaginary part of the self energy for the decay process. $\mathrm{Re}\bar{\Sigma}^{\mathrm{ret}}$ is the corresponding real part of the self energy (cf. e.g. [4] for details). τ as given in (8) is shown in Fig. 3 as the full line for the case of a Δ in vacuum. Obviously the two curves depicted in Fig. 3 are strikingly different. It is, however, interesting to note that both curves have an intuitive interpretation: If a resonance with arbitrary invariant mass is considered as a "real particle", it is clear that its life time increases near threshold due to the limited phase-space available for the decay products. This consideration suggests formula (6) corresponding to the dashed line in Fig. 3. If, however, the resonance is considered as a quantum mechanical transient state the uncertainty principle applies. Hence the more the invariant mass of the resonance deviates from its pole mass the less time it is allowed to live. Qualitatively this is the essence of (8) depicted as the full line in Fig. 3.

Since a resonance is not an asymptotically stable but an intermediate state we regard the second life time prescription (8) as the more reasonable one. Before we will generalize this formula to the case where one has to deal with more than one decay channel we shall discuss some consistency requirements which has to be fulfilled by any life time prescription.

3. Consistency requirements

No matter what formula one uses for the partial life times with respect to various decay channels, one has to make sure that physically measurable cross sections remain the same. Suppose that a resonance has two decay channels a and b. The survival probability is then given by $p(t) \sim e^{-t/\tau_a} \, e^{-t/\tau_b}$, i.e. the total life time is given by

$$\frac{1}{\tau_{\mathrm{tot}}} = \frac{1}{\tau_a} + \frac{1}{\tau_b} \qquad \Longleftrightarrow \qquad \tilde{\Gamma}_{\mathrm{tot}} = \tilde{\Gamma}_a + \tilde{\Gamma}_b \tag{12}$$

where for later convenience we have introduced $\tilde{\Gamma}_i := 1/\tau_i$. (Of course, for the standard $1/\Gamma$ prescription for the life time, $\tilde{\Gamma}$ coincides with Γ.) The probability that the resonance at some stage has decayed into channel a is given by $\tilde{\Gamma}_a/\tilde{\Gamma}_{\mathrm{tot}}$. This probability is nothing but the branching ratio $B = \Gamma_a/\Gamma_{\mathrm{tot}}$ and we have to make sure that this branching ratio remains the same no matter what prescription we use for the total and partial life times, i.e.

$$\frac{\tilde{\Gamma}_i}{\tilde{\Gamma}_{\mathrm{tot}}} = B = \frac{\Gamma_i}{\Gamma_{\mathrm{tot}}} \,. \tag{13}$$

The reason why we insist on that constraint is the following: Only if (13) holds we can ensure that (measurable) multi-step processes remain unchanged for different life time prescriptions, e.g.

$$\frac{d\sigma_{NN\to NN\pi}}{d\Omega} \approx \frac{d\sigma_{NN\to N\Delta}}{d\Omega} \underbrace{\frac{\Gamma_{\Delta\to N\pi}}{\Gamma_{\text{tot}}}}_{=B} \tag{14}$$

As a second constraint we demand that the mass averaged life time $\langle\tau\rangle$ which we defined in (4) approximately stays the same for any life time prescription (if the width is not extremely large):

$$\langle\tau\rangle = \int_0^\infty \frac{dE}{2\pi}\,\mathcal{A}(E)\,\tau(E) \approx \frac{1}{\Gamma_{\text{on-shell}}} \approx \int_0^\infty \frac{dE}{2\pi}\,\mathcal{A}(E)\,\frac{1}{\Gamma(E)}\,. \tag{15}$$

4. Life times and cross sections for off-shell states

We now demand that our preferred life time prescription (8) generalizes to the total life time for the case of several decay channels:

$$\tau_{\text{tot}} = \frac{1}{2}\mathcal{A}\,(1-K) =: 1/\tilde{\Gamma}_{\text{tot}} \tag{16}$$

where now in the formulae (10) and (11) the total width enters. Having specified $\tilde{\Gamma}_{\text{tot}}$ we get all partial $\tilde{\Gamma}_{\text{part}}$ — and thus all partial life times — on account of the consistency condition (13) yielding

$$\tau_{\text{part}} = \frac{\Gamma_{\text{tot}}}{\Gamma_{\text{part}}}\,\tau_{\text{tot}}\,. \tag{17}$$

With (16) and (17) all life times can be calculated. Concerning the consistency requirements discussed in the last section it is clear that by construction (13) is fulfilled. Choosing the total life time according to (16) also ensures (15) as discussed in [4].

As already pointed out (inelastic) scattering events with a resonance as an incoming state can also be understood as resonance decay processes. Clearly one can attribute a scattering cross section $d\tilde{\sigma}/d\Omega$ and also a partial life time to this process and both are of course related to each other:

$$\frac{d\tilde{\sigma}}{d\Omega}(M) \sim \frac{1}{\tau_{\text{scatt}}(M)} = \frac{\Gamma_{\text{scatt}}(M)}{\Gamma_{\text{tot}}(M)\,\tau_{\text{tot}}(M)}\,. \tag{18}$$

Using (7) we end up with

$$\frac{d\tilde{\sigma}}{d\Omega}(M) = \frac{1}{\Gamma_{\text{tot}}(M)\,\tau_{\text{tot}}(M)}\,\frac{d\sigma}{d\Omega}(M) \tag{19}$$

where on the left hand side we find the proper cross section for this process while on the right hand side we have the one which is calculated according to Feynman's rules as if the resonance was an asymptotic state with only one invariant mass M (recall the discussion after eq. (7)). The prefactor, i.e. the inverse of

$$r(M) := \Gamma_{\rm tot}(M)\,\tau_{\rm tot}(M) \tag{20}$$

which obviously renormalizes the naively calculated cross section is depicted in Fig. 4.

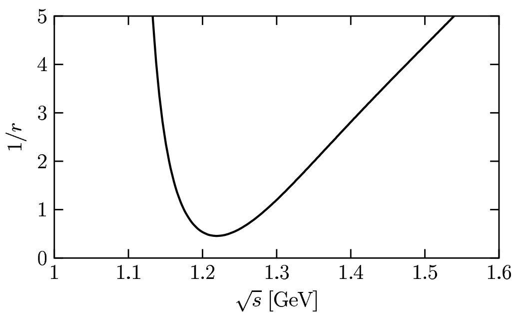

Fig. 4. Renormalization factor for all cross sections with an incoming resonance. See main text for details.

Having worked out the expressions for proper life times and cross sections we can ask about observable consequences of our prescriptions as compared to the standard recipe (6) [6]. For example one can study the pion absorption on nuclei as a function of the incident pion energy. The underlying microscopic process of pion absorption is $\pi N_1 N_2 \to \Delta N_2 \to N_3 N_4$. Since the absorption process happens via the excitation of a Δ one might expect changes in the absorption cross section if the life time prescription is changed. As shown in Fig. 5 the standard prescription (dashed line) yields less absorption as compared to the experimental results. If the life time is changed from the dashed to the full line in Fig. 3 one observes that the Δ's near the peak position live longer. Hence one would naively expect that the chance for the Δ to get hit by a nucleon grows. This would increase the absorption cross section. Indeed changing *only* the life time yields the dotted curve in Fig. 5. However, as pointed out above we also have to change the cross section for $\Delta N \to NN$. For a Δ near the peak position Fig. 4 indicates that the cross section is reduced. This reduces the Δ absorption and therefore also the pion absorption cross section. Hence our final result, the full line in Fig. 5, does not deviate much from the standard approach. Therefore, changing life time and cross section in a consistent way cannot explain the deviation between theory and experiment. We note in passing that the data can be explained if an additional absorption channel is included, namely $\pi N_1 N_2 N_3 \to \Delta N_2 N_3 \to N_4 N_5 N_6$, i.e. absorption of a Δ on two nucleons (for details we refer to [6] and references therein).

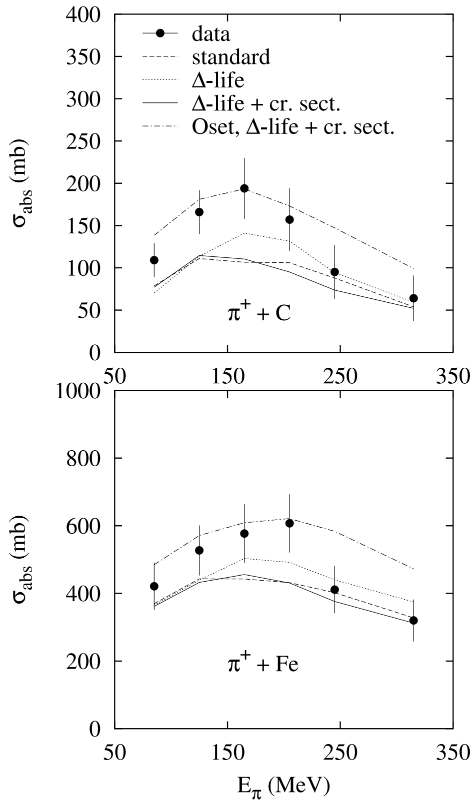

Fig. 5. Pion absorption on nuclei as a function of the pion energy for different prescriptions for life times and cross sections [6].

5. Further remarks and summary

Before summarizing we would like to put the work presented here in the context of recent developments in that area. We tried to present here rather intuitive arguments to justify our final results for life times and cross sections of broad resonances. In contrast to that the papers [4, 5] present a formal derivation of the same results by starting out from the quantum field theoretical Kadanoff-Baym equations and performing a first order gradient expansion *without* applying the commonly used quasi-particle approximation. In addition to the results presented here also the propagation of off-shell states is discussed in [5]. All these results are not settled issues yet but under present discussion. Related works with partly different results can be found in [9–11]. Basically all the works [4, 5, 9–11] contain comments where they differ or agree with each other.

To summarize we have argued that for the description of resonances as test particles in transport simulations the proper life time scales with the spectral function and not with the inverse width. This result is in line with the work of Danielewicz and Pratt [8] but in contrast to the commonly used $1/\Gamma$ prescription. Using our prescription for the life time we have shown that one has to change accordingly the cross sections for all scattering events with a resonance in an entrance channel. It turned out [6] that changing consistently life time and cross section diminishes the possibility to pin down observable effects of these changes. Clearly such an analysis concerning observable effects has to be extended in the future.

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References

1. W. Cassing, V. Metag, U. Mosel and K. Niita, Phys. Rept. **188** (1990) 363; S.A. Bass *et al.*, Prog. Part. Nucl. Phys. **41** (1998) 225.
2. M. Post, S. Leupold and U. Mosel, Nucl. Phys. **A689** (2001) 753.
3. G. Wolf, these proceedings.
4. S. Leupold, Nucl. Phys. **A695** (2001) 377.
5. S. Leupold, Nucl. Phys. **A672** (2000) 475.
6. A.B. Larionov, M. Effenberger, S. Leupold and U. Mosel, nucl-th/0107031.
7. Particle Data Group, D.E. Groom et al., E. Phys. J. **C15** (2000) 1.
8. P. Danielewicz and S. Pratt, Phys. Rev. **C53** (1996) 249.
9. Y.B. Ivanov, J. Knoll and D.N. Voskresensky, Nucl. Phys. A **657** (1999) 413; Nucl. Phys. A **672** (2000) 313;
   J. Knoll, Y.B. Ivanov and D.N. Voskresensky, Annals Phys. **293** (2001) 126.
10. W. Cassing and S. Juchem, Nucl. Phys. A **665** (2000) 377; Nucl. Phys. A **672** (2000) 417; Nucl. Phys. A **677** (2000) 445.
11. M. Effenberger, E.L. Bratkovskaya and U. Mosel, Phys. Rev. C **60** (1999) 044614;
    M. Effenberger and U. Mosel, Phys. Rev. C **60** (1999) 051901.