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Self-consistent antikaon dynamics in isospin-asymmetric nuclear medium

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Abstract. We investigate properties of antikaons and hyperon resonances in isospin-asymetric nuclear medium, using a self-consistent, covariant scheme based on vacuum antikaon-nucleon scattering amplitude.

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1. Introduction

¿From the time of first suggestion that kaons may condense in neutron stars [1], study of their properties attracted much interest [2–12]. Understanding kaon behavior in nuclear medium is also necessary for description of kaonic atoms [13, 14] and subtreshold production of kaon in heavy-ion reactions [15]. The latter requires transport model calculations using codes which can incorporate consistently particles with finite width, which are under development [16–19].

Softening of the antikaon spectral function in the nuclear medium was already anticipated from the K-matrix analyses of the antikaon-nucleon scattering [20], which predicted considerable attraction in the subtreshold scattering amplitudes. For reliable calculation of the in-medium antikaon spectral function it is necessary to have an improved understanding of vacuum antikaon-nucleon scattering (especially for the subtreshold region).

Treatment of the antikaon-nucleon scattering is made rather involved by the open inelastic $\pi\Sigma$ and $\pi\Lambda$ channels, as well as by the influence of the s-wave $\Lambda(1405)$, p-wave $\Sigma(1385)$ and d-wave $\Lambda(1520)$ resonances close to the threshold. Furthermore, relatively large errors in the low-energy data make the subtreshold extrapolation uncertain. It is then essential to use theoretical constraints (chiral symmetry, causality, covariance) while constructing the scattering amplitudes. Two

approaches ([21] and [22]) in this direction still gave markedly different subtreshold scattering amplitudes, triggering a recent work [23] which systematically includes s-, p- and d-waves with total angular momentum up to $J = 3/2$. The amplitudes obtained in that work are approximately crossing symmetric (the kaon-nucleon and antikaon-nucleon amplitudes match closely in the subtreshold region) and are consistent with chiral symmetry and covariance.

For calculation of in-medium antikaon spectral function a self-consistent scheme is necessary [11], since the effect of the modified antikaon propagation on in-medium scattering process (for example on the $\Lambda(1405)$) is very important. Recently a novel, covariant framework was developed [24] in which the self consistency was implemented in terms of the vacuum meson-nucleon scattering amplitudes. The method correctly takes into account the in-medium mixing of s-, p- and d-waves, and was used to study antikaon and hyperon-resonance properties in isospin-symmetric nuclear medium [24].

In section 2. we extend the formalism to isospin-asymmetric medium with arbitrary neutron and proton desities. In section 3. we present numerical results corresponding to neutron excess typical for lead and also for neutron matter.

2. Formalism

We first briefly recall the self consistent and covariant many-body framework introduced in [24]. The vacuum on-shell antikaon-nucleon scattering amplitude is

$$\begin{aligned}\langle \bar K^j(\bar q)\, N(\bar p)|\, T\, |\bar K^i(q)\, N(p)\rangle &= (2\pi)^4\, \delta^4(q+p-\bar q-\bar p) \\ &\quad \times \bar u(\bar p)\, T^{ij}_{\bar K N\to \bar K N}(\bar q,\bar p;q,p)\, u(p)\,, \end{aligned} \tag{1}$$

where the delta-function guarantees energy-momentum conservation and $u(p)$ is the nucleon isospin-doublet spinor. In quantum field theory the scattering amplitudes $T_{\bar K N}$ follow as solution of the Bethe-Salpeter matrix equation

$$\begin{aligned} T(\bar k,k;w) &= K(\bar k,k;w) + \int \frac{d^4 l}{(2\pi)^4}\, K(\bar k,l;w)\, G(l;w)\, T(l,k;w)\,, \\ G(l;w) &= -i\, S_N(\tfrac{1}{2}\, w+l)\, D_{\bar K}(\tfrac{1}{2}\, w-l)\,, \\ w &= p+q=\bar p+\bar q\,, \quad k=\frac{1}{2}\,(p-q)\,, \quad \bar k=\frac{1}{2}\,(\bar p-\bar q)\,, \end{aligned} \tag{2}$$

in terms of the Bethe-Salpeter kernel $K(\bar k,k;w)$, the free space nucleon propagator $S_N(p) = 1/(\not p - m_N + i\,\epsilon)$ and kaon propagator $D_{\bar K}(q) = 1/(q^2 - m_K^2 + i\,\epsilon)$.

The scattering process is readily generalized from the vacuum to the nuclear matter case. In compact notation:

$$\mathcal{T} = \mathcal{K} + \mathcal{K}\cdot\mathcal{G}\cdot\mathcal{T}\,, \quad \mathcal{T} = \mathcal{T}(\bar k,k;w,u)\,, \quad \mathcal{G} = \mathcal{G}(l;w,u)\,, \tag{3}$$

where the in-medium scattering amplitude $\mathcal{T}(\bar k,k;w,u)$ and the two-particle propagator $\mathcal{G}(l;w,u)$ depend now on the 4-velocity u_μ characterizing the nuclear matter

frame. For nuclear matter moving with a velocity $\vec{v}$:

$$u_\mu = \left(\frac{1}{\sqrt{1-\vec{v}^{\,2}/c^2}}, \frac{\vec{v}/c}{\sqrt{1-\vec{v}^{\,2}/c^2}} \right) , \quad u^2 = 1 . \tag{4}$$

The in-medium two-particle propagator $\mathcal{G}$ is given by

$$\begin{aligned}
&\Delta S_N(p,u) = 2\,\pi\, i\,\Theta\Big(p\cdot u\Big)\,\delta(p^2-m_N^2)\,(\not{p}+m_N)\,\Theta\big(k_F^2+m_N^2-(u\cdot p)^2\big)\,,\\
&\mathcal{S}_N(p,u) = S_N(p)+\Delta S_N(p,u)\,, \quad \mathcal{D}_{\bar K}(q,u) = \frac{1}{q^2-m_K^2-\Pi_{\bar K}(q,u)}\,,\\
&\mathcal{G}(l;w,u) = -i\,\mathcal{S}_N(\tfrac{1}{2}\,w+l,u)\,\mathcal{D}_{\bar K}(\tfrac{1}{2}\,w-l,u)\,,
\end{aligned} \tag{5}$$

where the Fermi momentum k_F is different for neutrons and protons.

The antikaon self energy $\Pi_{\bar K}(q,u)$ is evaluated self consistently in terms of the relevant in-medium scattering amplitudes

$$\Pi_{\bar K}(q,u) = 2\,\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4}\, i\,\Delta S_N(p,u)\,\bar{\mathcal{T}}_{\bar K N}\big(\tfrac{1}{2}\,(p-q),\tfrac{1}{2}\,(p-q);p+q,u\big). \tag{6}$$

The in-medium scattering amplitude $\bar{\mathcal{T}}$ is defined with respect to the free-space interaction kernel K, since we study only the effect of change in the medium propagation of nucleons and antikaons:

$$\hat{\mathcal{T}} = K + K\cdot\mathcal{G}\cdot\hat{\mathcal{T}} = T + T\cdot\Delta\mathcal{G}\cdot\hat{\mathcal{T}}\,, \quad \Delta\mathcal{G}\equiv\mathcal{G}-G\,. \tag{7}$$

The vacuum scattering amplitude can be decomposed systematically into covariant projectors $Y_n^{(\pm)}(\bar q,q;w)$ with good angular momentum and parity [23]. Including the leading terms with $J=1/2$ and $J=3/2$ (as was done in the numerical part of ref. [23]), means considering s-, p- and d-waves (the latter only with $J=3/2$). The relevant projectors are: for s-wave $Y_0^{(+)}$, for p-wave with $J=1/2$ $Y_0^{(-)}$, for p-wave with $J=3/2$ $Y_1^{(+)}$ and for d-wave with $J=3/2$ $Y_1^{(-)}$, given by

$$\begin{aligned}
Y_0^{(\pm)}(\bar q,q;w) &= \frac{1}{2}\left(\frac{\not{w}}{\sqrt{w^2}}\pm 1\right)\,,\\
Y_1^{(\pm)}(\bar q,q;w) &= \frac{3}{2}\left(\frac{\not{q}}{\sqrt{w^2}}\pm 1\right)\left(\frac{(\bar q\cdot w)(w\cdot q)}{w^2}-(\bar q\cdot q)\right)\\
&\quad -\frac{1}{2}\left(\not{\bar q}-\frac{w\cdot\bar q}{w^2}\not{w}\right)\left(\frac{\not{w}}{\sqrt{w^2}}\mp 1\right)\left(\not{q}-\frac{w\cdot q}{w^2}\not{w}\right).
\end{aligned} \tag{8}$$

In the presence of nuclear matter the scattering amplitude decomposition is more complicated, due to appearance of another 4-vector, the medium 4-velocity u_μ. The generalization of the projector algebra to this case has been worked out in ref. [24]. The physical reason for the presence of larger number of projectors is that

the partial waves mix, thus necessitating introduction of the off-diagonal matrix elements to the diagonal ones of eq. (8).

The mixing of partial waves happens in the following way. If the total momentum of the nucleon and antikaon is zero in the rest frame of the medium, there is no mixing at all. In this case the rotational symmetry of the medium, with respect to an observer who is at rest in it, assures that the total angular momentum J is a good quantum number which, together with parity, prevents mixing of partial waves.

If the observer moves with respect to the medium, i.e. $\vec{w} \neq 0$ in the rest frame of the medium, not all components of the total angular momentum are good quantum numbers, since only rotations about an axis defined by the momentum of the observer are still a symmetry. This means that only helicity remains a good quantum number and that states with different helicity do not mix (actually, what matters in our case is only the absolute value of the helicity). This picture is borne out by the projector algebra worked out in [24], where it was shown that the algebra decomposes into two subalgebras. One corresponds to elements of a 2×2 matrix, where the diagonal elements are the amplitudes of the two partial waves with $J = 3/2$. This obviously describes the mixing of the helicity-3/2 states. The other subalgebra, after elimination of zero entries, corresponds to a 4×4 matrix, whose diagonal elements are the amplitudes of all four partial waves. This describes the mixing of helicity-1/2 states, present in all four considered partial waves.

We now turn to considering the effects of isospin asymmetry of the nuclear medium. Consequently, we have to evaluate separately the self energy of the neutral antikaon and the negative kaon, using appropriate in-medium scattering amplitudes in expression (6). In each case there are two integrations, over the neutron and proton Fermi sea.

We chose to work in the particle bases, as opposed to the isospin one. In the former the loop integrals, containing as integrand the propagators of a nucleon and an antikaon, are obvously diagonal. There is, of course, mixing in some vacuum scattering amplitudes, which are given as:

$$T_{p\bar{K}^0\to p\bar{K}^0} = T_{nK^-\to nK^-} = T^{(1)}, \;\; T_{n\bar{K}^0\to n\bar{K}^0} = T_{pK^-\to pK^-} = \frac{1}{2}(T^{(0)} + T^{(1)}),$$
$$T_{pK^-\to n\bar{K}^0} = T_{n\bar{K}^0\to pK^-} = \frac{1}{2}(T^{(0)} - T^{(1)}), \qquad (9)$$

where $T^{(0)}$ ($T^{(1)}$) is the isospin-0 (isospin-1) amplitude.

3. Numerical results

The computational scheme outlined in the previous section requires as input only the vacuum antikaon-nucleon scattering amplitudes for s-, p- and d-waves. The amplitudes recently obtained in ref. [23] are especially well suited for this purpose, since they systematically incorporate constraints from chiral symmetry, crossing

symmetry and causality, and describe the antikaon-nucleon scattering quantitatively up to kaon laboratory momentum of 500 MeV (qualitatively up to 1000 MeV).

We use an iterative procedure to solve the eq. (7), starting from the calculation of antikaon self energies from vacuum amplitudes. Then we calculate the loop integrals, i.e. $\Delta\mathcal{G}$, which is used to compute the in-medium scattering amplitudes. The elf energies are then computed from the latter ones and the procedure is repeated until convergence is achieved (4–5 iterations are enough).

3.1. Antikaon spectral function in nuclear medium

In fig. 1 we show the antikaon spectral functions in neutron matter of density $\rho = 0.17$ fm^{-3}, for different kaon momenta. The effect of the medium is more pronounced on the $\bar{K}^0$, since protons are not present and thus the K^- can not produce the $\Lambda(1405)$.

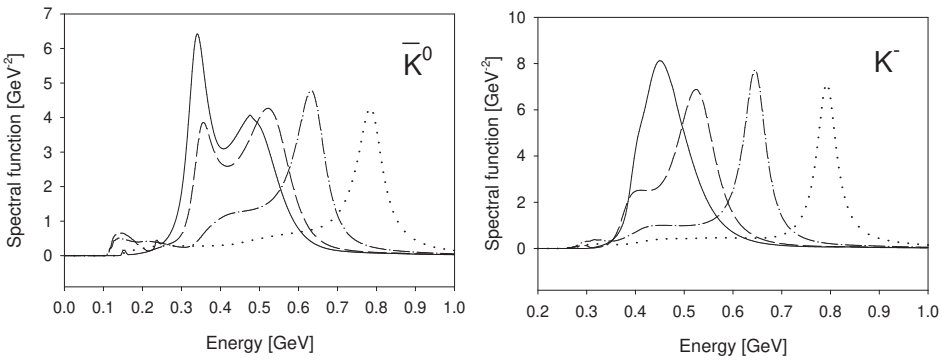

Fig. 1. Antikaon spectral functions in neutron matter of density $\rho = 0.17\,\mathrm{fm}^{-3}$, for different momenta: 0 (full line), 200 MeV (dashed line), 400 MeV (dash-dot line), 600 MeV (dotted line).

In fig. 2 the antikaon spectral functions are shown for the case of two lead nuclei superimposed, i.e. for neutron and proton densities double that of the central part of lead nucleus (we used the following Fermi momenta: for protons $p_{F_p} = 310.8$ MeV and for neutrons $p_{F_n} = 355.8$ MeV). We observe a dramatic broadening at small to intermediate momenta (up to about 500 MeV),which means that the quasiparticle approximation is inadequate. Even at larger momenta, 600–700 MeV, the broadening is quite significant and the spectral function is nonzero (although small) in the low-energy region. Again, the neutral antikaon is more affected, but the difference is not pronounced. Both spectral functions show nonzero support at small energy (starting from around 90 MeV) and that may have non-negligible effects in heavy-ion collisions.

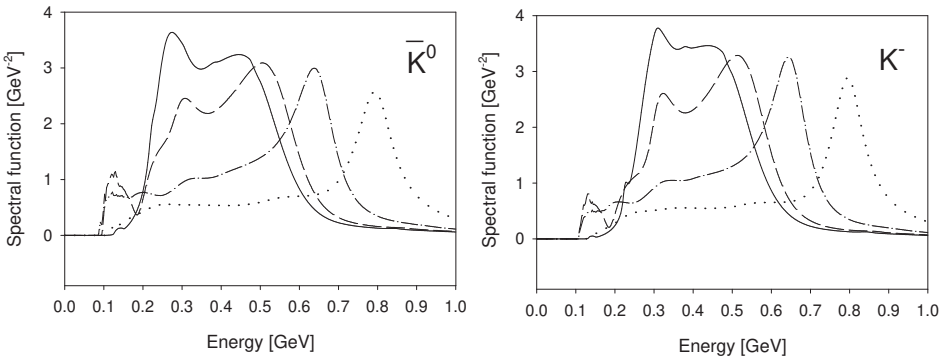

Fig. 2. Antikaon spectral functions in the medium with proton and neutron density double that of the central part of lead nucleus, for different momenta: 0 (full line), 200 MeV (dashed line), 400 MeV (dash-dot line), 600 MeV (dotted line).

3.2. Hyperon resonances in nuclear medium

¿From the in-medim scattering amplitudes we can infer the changes in the hyperon resonances playing a role in the considered s-, p- and d-waves. The $\Lambda(1405)$ plays a dominant role in the $I = 0$ s-wave and is modified considerably in isospin-symmetric medium [24]. In fig. 3 we show the real and imaginary part of the corresponding scattering amplitude in neutron matter and, for comparison, vacuum.

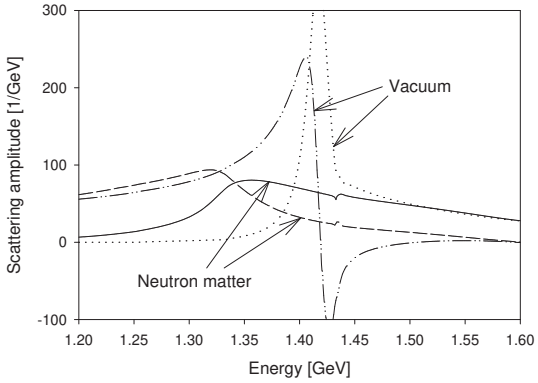

Fig. 3. The $\Lambda(1405)$ at rest in neutron matter of density $\rho = 0.17\,\mathrm{fm}^{-3}$. The full (dashed) line shows the imaginary (real) part of the relevant scattering amplitude, while the dotted (dash-dot-dot) line shows the imaginary (real) part of the amplitude in vacuum.

For lead (where the neutron number is 50% higher than the proton number) the effect on $\Lambda(1405)$ is quite similar to the one in isospin-symmetric medium with the same nucleon density, as shown in ref. [24]. However, increasing the proton and neutron density by a factor of two produces even more pronounced broadening than the one shown in fig. 3.

It may be of interest to look at the splitting of the $\Sigma(1385)$ in isospin-asymmetric medium. Figs. 4 and 5 show the imaginry part of the corresponding amplitudes for zero total momentum (fig. 4) and 400 MeV total momentum (fig. 5).

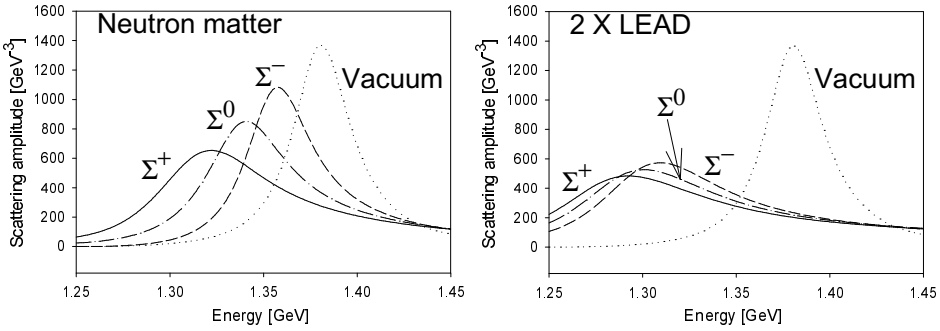

Fig. 4. The $\Sigma(1385)$ at rest in neutron matter of density $\rho = 0.17\,\mathrm{fm}^{-3}$ (left figure) and nuclear matter double the lead density (right figure). The full line shows the imaginary part of the scattering amplitude corresponding to the Σ^+, dashed line to Σ^- and dash-dot line to Σ^0. The vacuum amplitude is shown for comparison by dotted line.

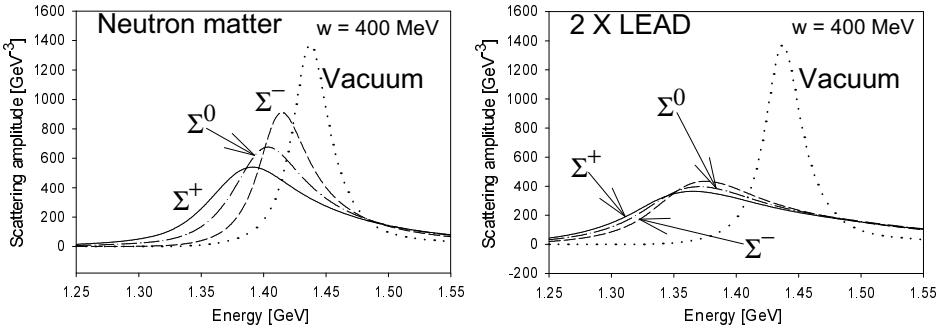

Fig. 5. The same as fig. 4, with only difference that the $\Sigma(1385)$ moves with momentum $|\vec{w}| = 400$ MeV in the medium.

The maxima of the resonances usually obey with good accuracy the vacuum relation $\sqrt{m_*^2 + \vec{w}^{\,2}}$, where m_* is the position for the resonance at rest. However,

as seen in fig. 5, the $\Sigma(1385)$ resonances are closer to each other at $|\vec{w}| = 400$ MeV, than the above relation would suggest.

The $\Sigma(1195)$ in neutron matter shows a splitting which is smaller than that of the $\Sigma(1385)$. The position of the maximum for the Σ^+ at rest is 1180 MeV, while the Σ^- is at 1192 MeV, with the Σ^0 being just in the middle between the charged states.

Strong medium modification characterizes also the other resonances playing part in the antikaon-nucleon scattering, for the case of presented densities. Exception is the $\Sigma(1690)$ which suffers only a minor broadening, probably as a consequence of small branching fraction into antikaon-nucleon channel.

In conclusion, we extended the recently developed method for self-consistent analysis of antikaons and hyperon resonances in isospin-symmetric nuclear medium to the isospin-asymmetric one. In neutron matter we observe a stronger medium modification of the $\bar{K}^0$ as compared to the K^-. The magnitude of the medium effects on the hyperon resonances is similar to the isospin-symmetric case, but with pronounced splitting of the $\Sigma(1385)$ and a smaller one for the $\Sigma(1195)$.

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